when slipping takes place. But from (3)

$$t_1 - t_2 = \frac{2 P R}{n r} = \frac{2 \times 300 \times 1\frac{1}{2}}{40 \times \frac{3}{4}} = 30 \text{ lbs.}$$

Therefore $$t_1 = 60 + 15 = 75$$
$$t_2 = 60 - 15 = 45$$

and $$\frac{t_1}{t_2} = 1{\cdot}5.$$

Thus, with the above conditions, slipping will not occur.

As a matter of experiment, the author finds that with such a smooth hub and an arc of contact of half a turn slipping takes place in riding up steep hills only when the spokes are initially slacker than is usual in ordinary tangent wheels.

Arc of Contact between Spokes and Hub.—The pair of spokes (fig. 347) is shown having an arc of contact with the hub of nearly two right angles. The arc of contact may be varied. For example, keeping the end a_1 fixed, the other end of the spoke may be moved from $a^1{}_1$ to $a^1{}_2$, or even further, so that the arc of contact may be as shown in figure 350. In this case there are five spoke ends left between the ends of one pair. In general, $4n + 1$ spokes must be left between the ends of the same pair, n being an integer.

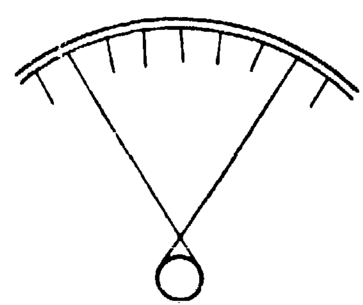


FIG. 350.

In this wheel, should one of the spokes break, a whole loop of wire must be removed. Of course the tendency to break is, as already shown, far less than in direct or tangent spokes of the usual type. If the arc of contact, however, is as shown in figure 347, and a pair of spokes are removed from the wheel, a great additional tension will be thrown on the spoke between the two vacant spaces. If the angle of contact shown in figure 350 be adopted, there will still remain five spokes between the two vacant spaces, so that the additional tension thrown on any single spoke will not be abnormally great.

Grooved Hubs.—The hub surface in contact with the spokes may be left quite smooth, with merely a small flange to preserve the spread of the spokes. The parts of the spokes wrapped round

the hub will lie in contact side by side (fig. 348) Should one break and be removed from the wheel, the remaining spokes in contact with the hub will close up the space vacated by the broken one. In putting in a new spoke they will have to be again separated. Spiral grooves may be cut round the hub, so that each spoke may lie in its own special groove, and if one breaks, the space will be left quite clear for the new spoke to replace it.

The grooves may be made so as to considerably increase the frictional grip on the nave. Figure 351 shows the section of a spoke in a groove, the spoke touching the sides, but not the bottom of the groove. It is pressed to the hub by a radial force, P, and the reactions R_1 and R_2 are at right angles to the side of the grooves. Figure 352 shows the corresponding force-triangle. The sum of the forces R_1 and R_2, between the spoke and the hub, may be increased to any desired multiple of P by making the angle between the sides of the groove sufficiently small, and the frictional grip will be correspondingly increased. If the angle of the sides of the grooves is such that $R_1 + R_2 = n\,P$, $n\,\mu$ must be used instead of μ in equations (4) and (5).

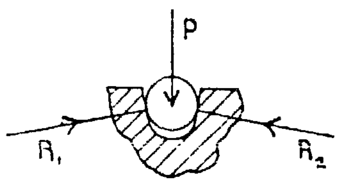


FIG. 351.

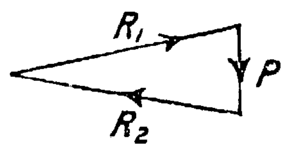


FIG. 352.

Example II.—If the spokes in the wheel in the above example lie in grooves, the sides of which are inclined 60°; find the driving effort that can be transmitted without slipping.

In this example the force-triangle (fig. 352) becomes an equilateral triangle, and $R_1 + R_2 = 2\,P$. Taking $\mu = \cdot 15$ and $\theta = \pi$ as before, $n\,\mu = \cdot 3$, and

$$log\frac{t_1}{t_2} = \cdot 4343 \times \cdot 3 \times 3 \cdot 141 = \cdot 4093,$$

from which, consulting a table of logarithms,

$$\frac{t_1}{t_2} = 2 \cdot 566.$$

But $t_1 + t_2 = 120$ lbs. Solving these two simultaneous simple equations, we get

$$t_1 = 86{\cdot}3$$
$$t_2 = 33{\cdot}7,$$

the driving effort is $t_1 - t_2 = 52{\cdot}6$ lbs.

Thus, the effect of the grooves inclined 60° is to nearly double the driving effort that can be transmitted.

252. **Spread of Spokes.**—If the spokes of a tension wheel all lay in the same plane, then, considering the rim fixed, any couple tending to move the spindle would distort the wheel, as shown in figure 353. The distortion would go on until the moment of the pull of the spokes on the hub was equal to the moment applied to the shaft. If the spindle remains fixed in position, any lateral force applied to the rim causes a deviation of its plane, the relative motion of the rim and spindle being the same as before; the wheel, in fact, wobbles. If the spokes are spread out at the hub (fig. 354), the rim being fixed and the same bending-moment being applied at the spindle, the tension on the spokes *A* at the bottom right-hand side, and on the spokes *B* at the top left-hand side, is decreased, and that on the spokes *C* at the left-hand bottom side, and on the spokes *D* at the right-hand top side is increased. This increase and diminution of tension takes place with a practically inappreciable alteration of length of the spokes, and therefore the wheel is practically rigid.

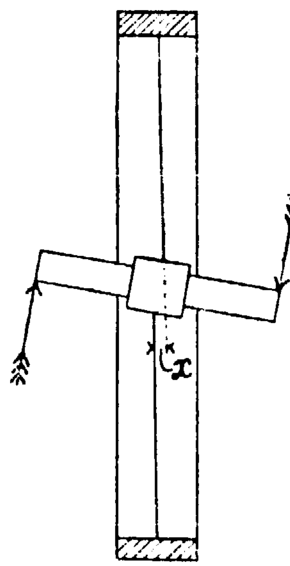


FIG. 353.

The lateral spreading of the spokes of a cycle wheel should be looked upon as *a means of connecting the hub rigidly to the rim*, rather than of giving the rim lateral stability relative to the hub. The rim must be of a form possessing initially sufficient lateral stability, otherwise it cannot be built up into a good wheel. The lateral components of the pulls of the spokes on the rim, instead of preserving the lateral stability of the rim, rather tend to destroy it. They form a system of equal and parallel forces, but alternately in opposite directions (fig. 355), and thus cause bending of the

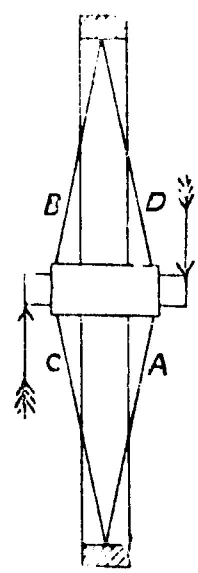


FIG. 354.

rim at right angles to its plane. If the rim be very narrow in the direction of the axis of the wheel, it may be distorted by the pull of the spokes into the shape shown exaggerated in figure 355.

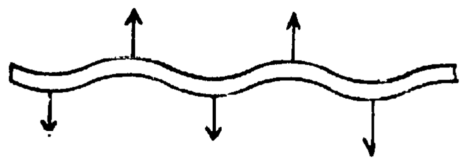


FIG. 355.

The 'Westwood' rim (fig. 373), on account of its tubular edges, is very strong laterally.

253. **Disc Wheels.**—Instead of wire spokes to connect the rim and hub, two conical discs of very thin steel plate have been used, the discs being subjected to an initial tension. It was claimed—and there seems nothing improbable in the claim—that the air resistance of

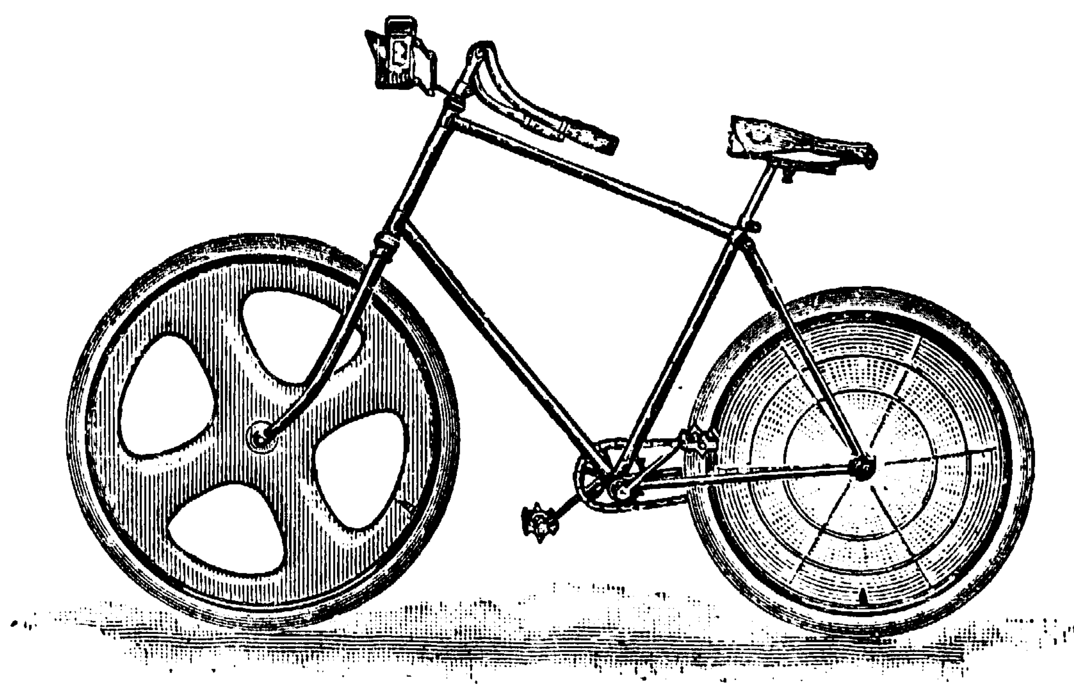


FIG. 356.

these wheels was less than that of wheels with wire spokes. Later, the Disc Wheel Company (Limited) made the front wheel of a Safety with four arms, as shown in figure 356.

Nipples.—The nipples used for fastening the ends of the spokes to the rim are usually of steel or gun-metal. Perhaps, on the whole, gun-metal nipples are to be preferred to steel, since they do not corrode, and being of softer metal than the spokes, they cannot cut into and destroy the screw threads on the spoke ends. Figure 357 is a section of an ordinary form of nipple which can be used for both solid and hollow rims, and figure 358 is an external view of the same nipple, showing its hexagonal external surface for screwing up. The hole in the nipple is not

tapped throughout its whole length, but the ends towards the centre of the wheel are drilled the full diameter of the spoke, so that the few extra screw threads left on the spoke to provide for the necessary adjustment are protected by the nipple. Figure 359 shows a square-bodied nipple, otherwise the same as that in figure 358.

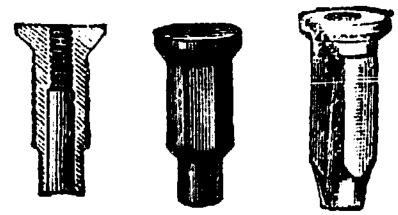


FIG. 357. FIG. 358. FIG. 359.

When solid rims are used, the nipple heads must be flush with the rim surface, so as not to damage the tyre; but when hollow rims are used, the nipple usually bears on the inner surface of the rim, and is therefore quite clear of the tyre. Figures 360 and 361 show forms of nipples for use with hollow rims, the screw thread of the spoke being protected by the latter nipple.

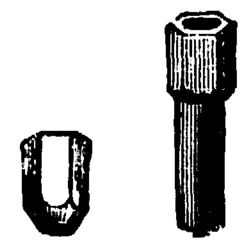


FIG. 360. FIG. 361

In rims of light section, such as the hollow rims in general use, the greatest stress is the *local* stress due to the screwing up of the spokes. With a very thin rim, which otherwise might be strong enough to resist the forces on it, the bearing surfaces of the nipples shown above are so small that the nipple would be actually pulled through the rim by the pull due to tightening the spoke. To distribute the pressure over a larger surface of the rim, small washers (fig. 362) may be used with advantage. With wood rims, washers should be used below the nipples, otherwise the wood may be crushed as the tension comes on the spokes.

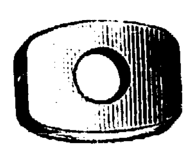


FIG. 362.

Figure 363 shows the form of steel nipple to be used with Westwood's rim when the spokes are attached, not at the middle, but at the sides of the rim. Figures 357-363 are taken from the catalogue of the Abingdon Works Company (Limited), Birmingham.

FIG. 363.

254. **Rims.**—We have already seen that the rim is subjected to a force of compression due to the initial pull on the spokes. Let us consider more minutely the stresses on the rim when the wheel is not supporting any external load. Let figure 364 be the elevation of a wheel with centre C, $A\ B$ being the chord between the ends of two adjacent spokes. Then

the stress-diagram (see figs. 334 and 335) of the structure will be a similar regular polygon, the pull on each spoke being repre-

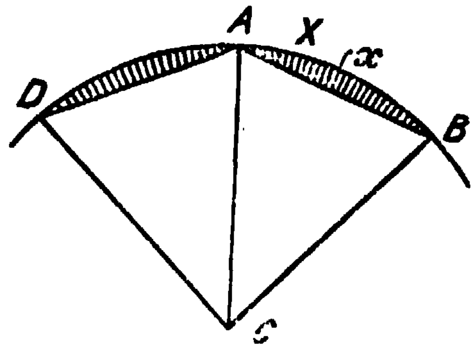


FIG. 364.

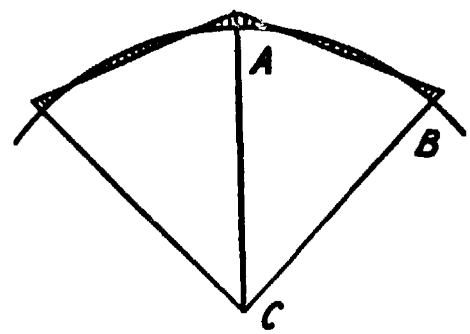


FIG. 365.

sented by the side and the compression on the rim by the radius of the polygon.

If the rim were polygonal, the axes of the rim and the compressive force on it would coincide, and the compressive stress would be equally distributed over the section. But since the rim is circular, its axis will differ from the axis of the compression, and there will be a bending-moment introduced. Since at any point X this bending-moment is equal to the product of the compression P into the distance x between the axis of the rim and the line of action of P, the bending-moment on the rim will be proportional to the intercept between the rim and the chord $A\,B$, formed by joining the ends of two adjacent spokes, provided that the bending-moment on the rim at the points where the spokes are fastened is zero. The shaded area (fig. 364) would thus form a bending-moment diagram. But if the rim initially had no bending stress on it, it is likely that at the points A and B the pull ot the spokes will tend to straighten the rim, and therefore a bending-moment, m, of some magnitude will exist at these points. The bending-moment at any point X will be diminished by the amount m, and the diagram will be as shown in figure 365, the bending-moments being of opposite signs at the ends of, and midway between, the spokes. From an inspection of figure 365, it is clear that in a wheel with 32 to 40 spokes, the bending-moment on the rim due to the compression will be negligibly small in comparison with the latter.

When the wheel supports a load the distribution of stress on the rim is much more complex, and a satisfactory treatment of

the subject is beyond the scope of the present work. The simplest treatment—which, however, the author does not think will give even rough approximations to the truth—will be to assume that the segments of the rim are *jointed* together at the points of attachment of the spokes. With this assumption, if the wheel supports a weight W, when the lowest spoke is vertical, the force-triangle at A, the point of contact with the ground, will be made up of the two compressions along the adjacent segments of the rim, and the pull on the vertical spoke plus the upward reaction of the ground, W. The rest of the stress-diagram will be as in the former case; consequently, if the pull on the vertical spoke is zero, that on the other spokes will be W; if the pull on the vertical spoke is t, that on the other spokes will be $(W + t)$.

When the two bottom spokes are equally inclined to the vertical, the lower rim segment is in the condition of a beam supported at the ends and carrying in the middle a load, W; therefore the bending-moment is $\frac{Wl}{4}$, l being the length of the rim segment.

The assumption made above does not agree, even approximately, with the actual condition of things in the continuous rim

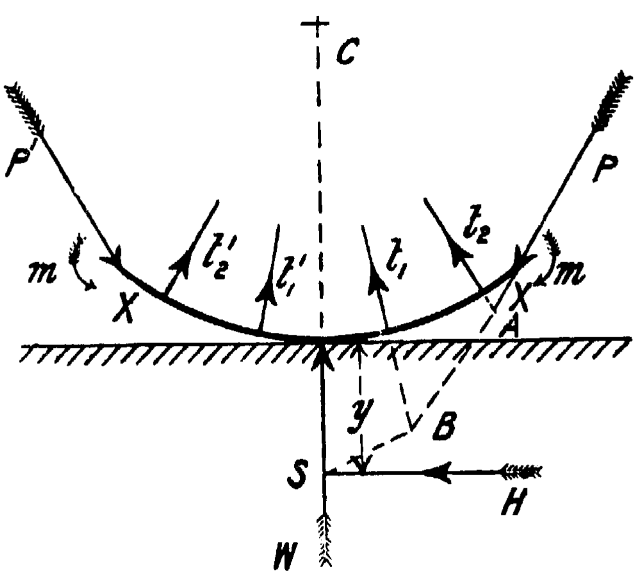


FIG. 366.

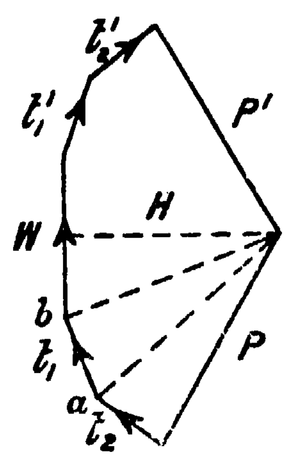


FIG. 367.

of a bicycle wheel. A general idea of the nature of the forces acting may be obtained from figure 366, which represents a small portion, XX, of the rim near the ground. This is acted on by the known force W, the upward reaction of the ground; by the un-

known forces t_1, t_2, . . . the pulls on the spokes directed towards the centre, C, of the wheel; by forces of compression, P, on the rim, unknown both in direction and magnitude; and by unknown bending-moments, m, at the section X. The portion of the rim considered is, therefore, somewhat in the condition of an inverted arch. If the forces P, t_1, t_2, . . . and the bending-moments, m, were known, the straining action at any point on the rim could be determined as follows: Figure 367 shows the force-polygon, on the assumption that the forces considered are symmetrically situated with regard to the vertical centre line. The horizontal thrust on the rim at its point of contact with the ground is H, the resultant of the forces P, t_1, t_2, . . . on one side of the vertical. This, however, acts at a point S, at a vertical distance y below the rim, determined as follows: Produce the lines of action of P and t_2 to meet at A; their resultant, which is parallel to $O\,a$ (fig. 367), passes through the point A. Draw, therefore, $A\,B$ parallel to $O\,a$, cutting the line of action of t_1 at B. Through B draw a line parallel to $O\,b$, giving the resultant of P, t_2, and t_1, and cutting the vertical through the point of contact at S. The rim at its point of contact with the ground is thus subjected to a compression H, and a bending-moment $m + H\,y$. To make the solution complete, the unknown forces P, t_1 and t_2 should be determined; this can be done by aid of the theory of elasticity.

Steel Rims.—Figure 368 shows a section of a rim for a solid tyre, figure 369 for a cushion tyre. The edges of the latter are

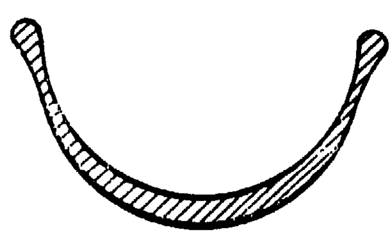


FIG. 368.

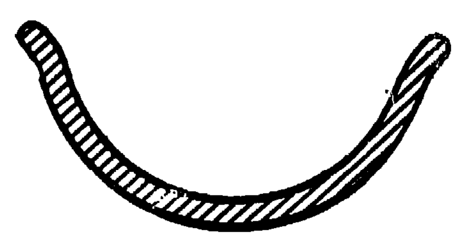


FIG. 369.

slightly bent over, so that the tyre when it bulges out on touching the ground will not be cut by the rim edge. Figure 370 shows a section of Warwick's hollow rim, which is rolled from one strip of steel bent to the required section, its edges scarfed, and brazed together. The part of the rim of smallest radius is thickened, so

that the local stresses due to the screwing-up of the spokes may be better resisted. Figure 371 shows the 'Invincible' rim which was

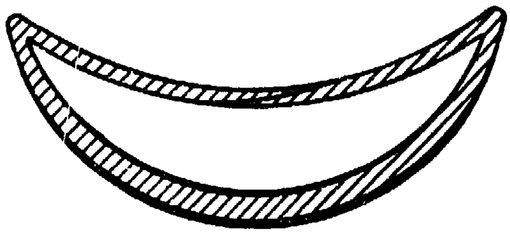


FIG. 370.

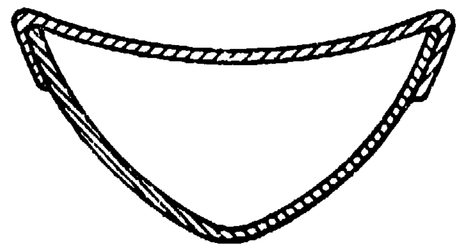


FIG. 371.

made by the Surrey Machinists Company, rolled from two distinct strips, the inner being usually much thicker than the outer. The strips

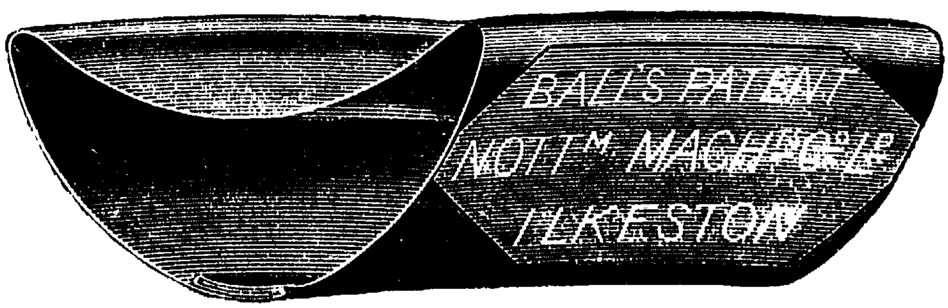


FIG. 372.

were brazed together right round the circumference. Figure 372 shows the Nottingham Machinists' hollow rim. In this the local strength for the attachment of the nipple is provided by folding over the plate from which the rim is made, so that four thicknesses are obtained. Figure 373 shows the 'Westwood' rim,

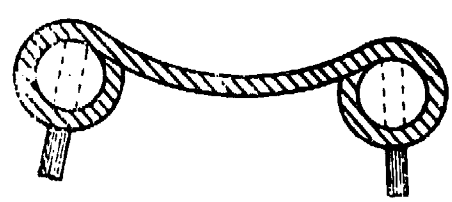


FIG. 373.

which is formed from one plate bent round at each edge to form a complete circle. The spokes can be attached at the edges of the rim as indicated, or at the middle of the rim in the usual way.

All the above rims are rolled to different sections to fit the different forms of pneumatic tyres. They are all made from straight strips of steel, and have, therefore, one joint in the circumference, the ends being brazed together. This joint, however carefully made, is always weaker than the rest of the rim, and adds to the difficulty of building the wheel true. The Jointless Rim Company roll each rim from a weldless steel ring, in somewhat the same way as railway tyres are rolled.

This rim, though perhaps more costly, is therefore much stronger weight for weight than a rim with a brazed joint.

Wood Rims.—The fact that the principal stress on the rim of a bicycle wheel is compression, and that, therefore, the material must be so distributed as to resist buckling or collapse, and not concentrated as in a steel wire, suggests the use of wood as a suitable material. Hickory, elm, ash, and maple are used. Two types are in use : in one the rim is made from a single piece of wood, the two ends being united by a convenient joint. Figure 374 shows the 'Plymouth' joint. The other type is a built-up

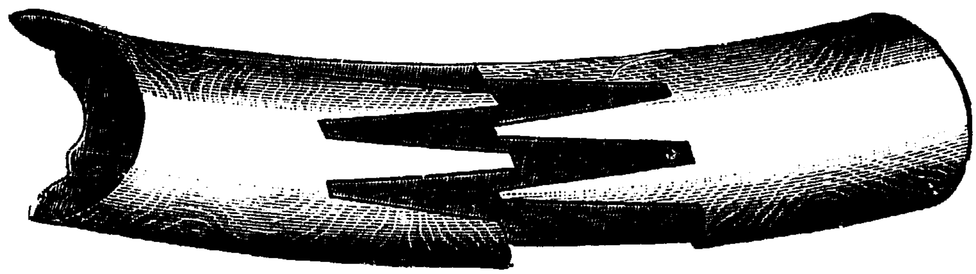


FIG. 374

rim composed of several layers of wood. Figures 375 and 376 show the 'Fairbank' laminated rim, for a solutioned tyre and for the Dunlop tyre respectively, the grain of each layer of wood running in an opposite direction to that next it. Each layer or ring is made with a scarfed joint, and the various rings are fastened together with marine glue under hydraulic pressure. The built-up rim is then covered with a waterproof linen fabric, and varnished.

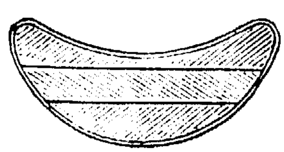


FIG. 375.

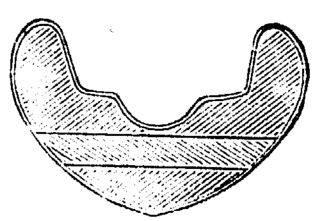


FIG. 376.

255. **Hubs.**—Figure 404 shows a section of the ordinary form of hub for a direct spoke-wheel, and figure 377 an external view of a driving hub. The hub proper in this is made as short as possible, and the spindle, with its adjusting cones, projects considerably beyond the hub, so as to allow the wheel to clear the frame of the machine.

Figure 378 shows a driving hub, in which the hub proper is extended considerably beyond the spoke flanges, and the ball-races

kept as far apart as possible. This hub is intended for tangent-spokes, the flanges being thinner than in figure 377.

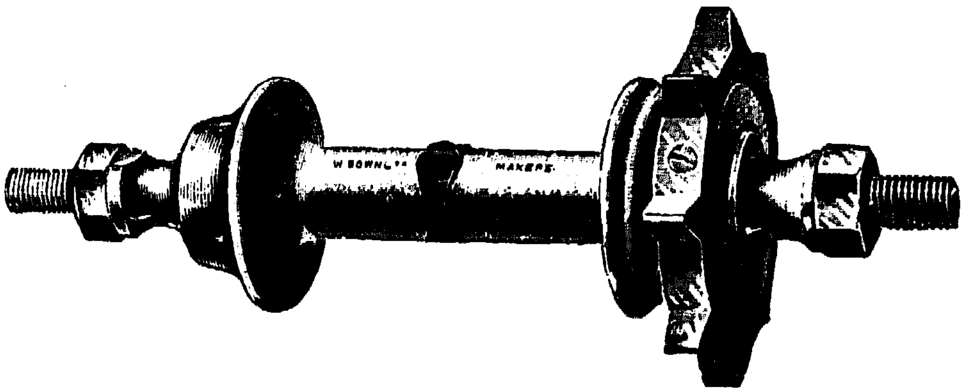


FIG. 377.

Hubs for direct-spokes are made either of gun-metal or steel ; tangent-spoke hubs should be invariably of steel, as the local

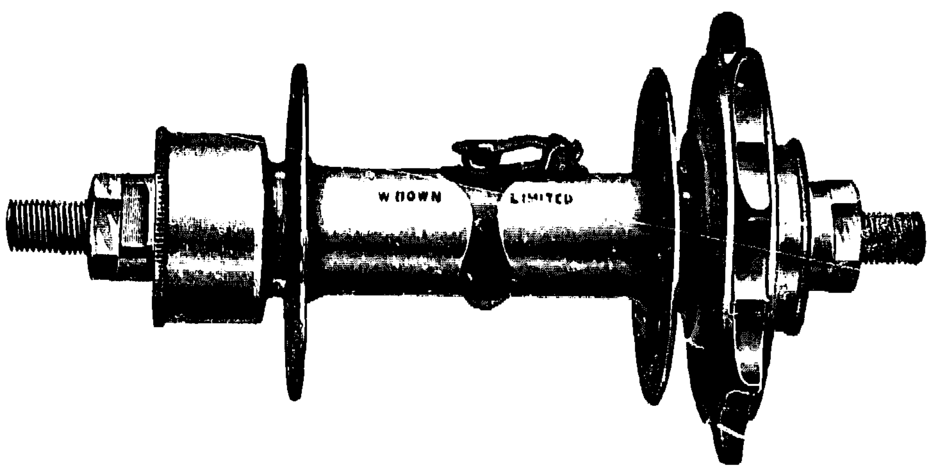


FIG. 378.

stress due to the pull of the spoke cannot be resisted by the softer metal.

Figure 379 shows a pair of semi-tangent hubs, as made by Messrs. W. A. Lloyd & Co., the flanges for the attachment of the

FIG. 379

hub in this case forming cylindrical drums instead of flat discs, as in figure 378. The spokes may leave the circumference of the

drum at any angle between the radius and the tangent, hence the name *semi-tangent*.

In all the hubs above described the adjusting cones are screwed on the spindle, and the hard steel cups are rigidly fixed to the hub. In the 'Elswick' hub (fig. 380), the adjusting cone

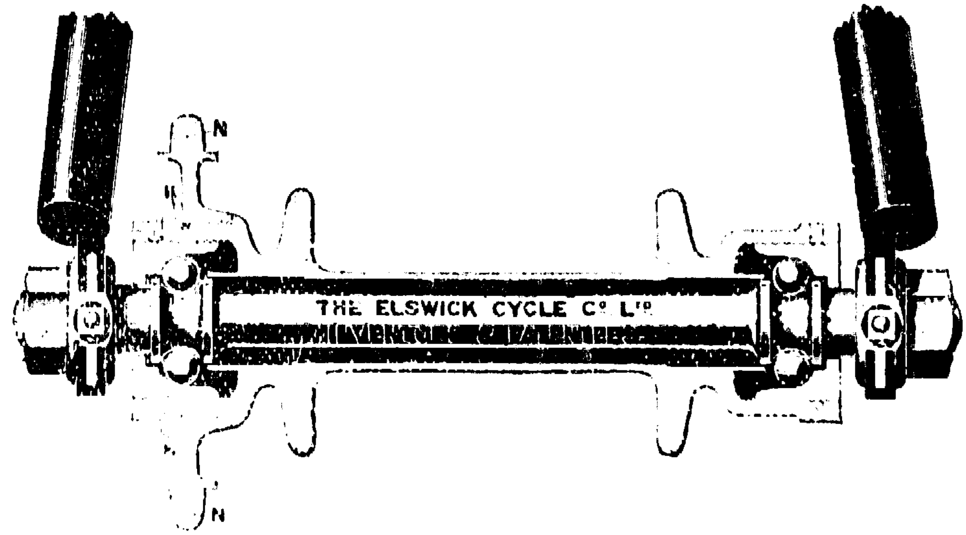


FIG. 380.

is screwed to the hub and the ball races on the spindle are rigidly fixed. One important advantage of this form of hub is that the clear space which must always be preserved between the fixed spindle and the rotating hub is of much smaller radius than in the others. The area by which dust and grit may enter the bearing

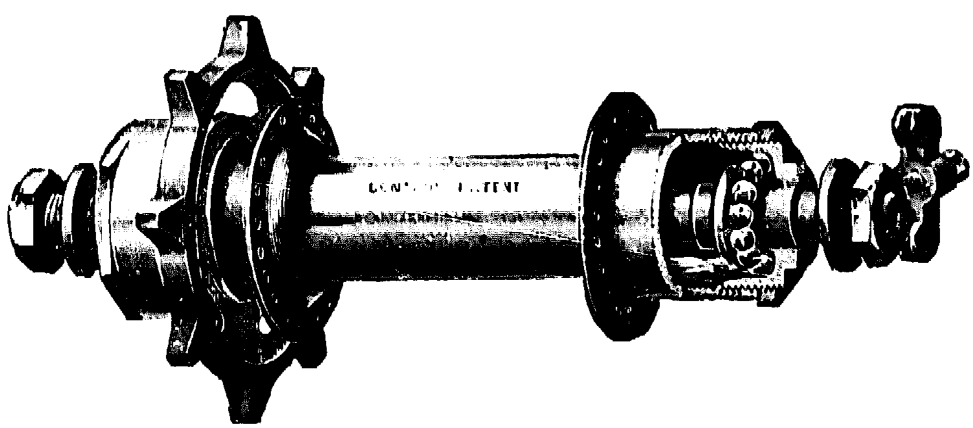


FIG. 381.

is smaller, the bearing should therefore be more dust-proof than the others. Another important feature is the fact that the hub is oil-retaining, and the balls may have oil-bath lubrication at the lowest point of their path. Figure 381 shows the 'Centaur' hub, also possessing dust-proof oil-retaining properties.

In recent years 'barrel' hubs of large diameter have been used, whereas the earlier hubs were made just large enough to clear the spindle inside. The 'Centaur' is an example of a barrel hub.

The best hubs are turned out of solid steel bar, the diameter of which must be as great as that of the flanges for the attachment of the spokes. To avoid this excessive amount of turning, the 'Yost' hub is made of two end pieces and a middle tube.

The hubs of Sharp's tangent wheel may, with advantage, be made of aluminium, since the pull of the spokes has not to be transmitted by flanges.

The 'Gem' hub, made by the Warwick and Stockton Company, has the hard steel cup screwed to the end of the hub. The balls lie between the cup and an inner projecting lip of the hub, so that they remain in place when the spindle is removed.

256. **Fixing Chain-wheel to Hub.**—The chain-wheel should not be fixed by a key or pin, as this will usually throw it slightly eccentric to the hub. In testing the resistance of the chain gearing of a Safety it is often noticed that the chain runs quite slack in some places and tight in others. This can only mean that the centres of the pitch-polygons of the chain-wheels do not coincide with the axes of rotation. The chain-wheel and the corresponding surface on the hub, being turned to an accurate fit, are often fastened by simply soldering. The temperature at which the solder melts is sufficiently low to prevent injury to the temper of the ball-races of the hub. Another method is to screw the chain-wheel, *N*, on the hub; the screw should then be arranged that the driving effort in pedalling ahead tends to screw the chain-wheel up against the projecting hub flange. This is done in the 'Elswick' hub (fig. 380). If the chain is at the right-hand side of the machine looking forwards, the screw on the chain-wheel should be right handed. During back-pedalling the driving effort will tend to unscrew the chain-wheel. This is counteracted by having a nut, *K*, with left-handed screw, screwed up hard against the chain-wheel. If the chain-wheel, *N*, tends to unscrew during back-pedalling, it will take with it the nut *K*, which will then be screwed more tightly against the wheel, and its further unscrewing prevented.

A method adopted by the Abingdon Company a few years ago was to have the chain-wheel and hub machined out to a polygonal surface of ten sides, and the wheel then soldered on.

257. **Spindles.**—The spindle, strictly speaking, is a part of the frame, and serves to transfer the weight of the machine and rider to the wheel. Let the spindle be connected to the frame at A and B (fig. 382), C and D be the points at which it rests on the

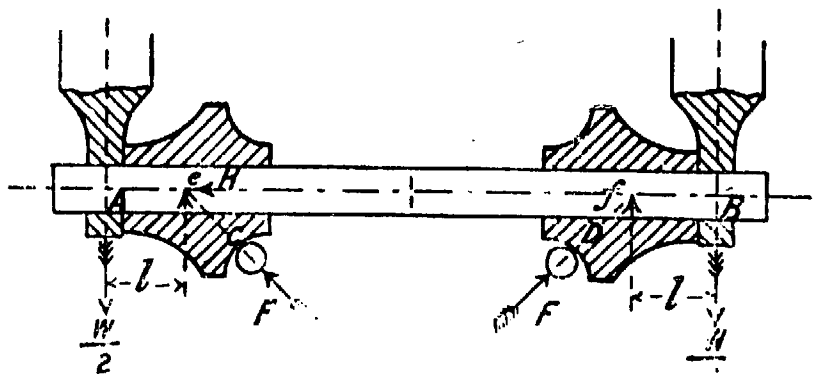


FIG. 382.

balls of the bearing, and W be the total load on the wheel. Then the spindle may be considered as a beam loaded at A and B with equal weights $\frac{W}{2}$, and supported at the points C and D; the direction of the forces of reaction F, at C and D, coinciding with the radii of the balls to their points of contact with their paths. Let e and f be the points at which the forces F cut the axis of the spindle; then F can be resolved into vertical and horizontal forces, $\frac{W}{2}$ and H respectively, acting through e. The horizontal forces, H, produce a tension on the part ef of the spindle, the remaining forces produce bending stresses. The spindle may thus be considered as a beam supported at e and f and loaded at A and B with equal weights, $\frac{W}{2}$. The bending-moment on any section between e and f is $\frac{Wl}{2}$, l being the distance Ae. It is evident that this bending-moment will be zero if the points A and e coincide, and will be greater the greater the distance Ae; hence the spindle in figure 378 is subjected to a far smaller bending stress than that in figure 377.

Example I.—In a bearing the distance $A\ e$ (fig. 382) is $\frac{7}{8}$ in., and the total weight on the wheel is 120 lbs., what is the necessary size of spindle, the maximum stress allowed being 10 tons per sq. in.?

The bending-moment on the spindle will be

$$\frac{120}{2} \times \frac{7}{8} = 52{\cdot}5 \text{ inch-lbs.}$$

Substituting in the formula $M = \frac{d^3}{10}f$ (sec. 94), we get

$$52{\cdot}5 = \frac{d^3}{10} \times 10 \times 2240,$$

that is

$$d^3 = {\cdot}0234, \text{ and } d = {\cdot}286 \text{ in.}$$

This gives the least permissible diameter of the spindle, that is, the diameter at the bottom of the screw threads.

Step.—The most convenient step for mounting a Safety bicycle is formed either by prolonging the spindle itself, or by forming a long tube on the outer nut that serves to fasten the spindle to the frame and lock the adjusting cone in position. If the length of this step be $1\frac{1}{2}$ in., W the weight of the rider, and if the rider in mounting the machine press on its outer edge, the bending-moment produced on the spindle will be $1\frac{1}{2}\ W$ inch-lbs.

Example II.—If $W = 150$ lbs., $M = 225$ inch-lbs.; substituting in the formula $M = Zf$,
we get

$$225 = \frac{d^3}{10} \times 10 \times 2240,$$

from which

$$d^3 = {\cdot}100 \text{ and } d = {\cdot}464 \text{ in.}$$

A common diameter for the spindle is $\frac{3}{8}$ in.; if the $\frac{3}{8}$ in. spindle resist the whole of the above bending-moment, the maximum stress on it will be much greater than 10 tons per sq. in.; it will be

$$\frac{{\cdot}464^3}{{\cdot}375^3} \times 10 = 18{\cdot}9 \text{ tons per sq. in.}$$

The tube from saddle-pin to driving-wheel spindle may take up some of the bending due to the weight on the step, in which case the maximum stress on the spindle may be lower than given above.

258. **Spring Wheels.**—Different attempts have been made to make the wheels elastic, so that vibration and bumping due to the unevenness of the road may not be communicated to the frame. One of the earliest successful attempts in this direction was the corrugated spokes used in the 'Otto' dicycle. These spokes, instead of being straight, were made wavy or corrugated, and of a harder quality of steel than used in the ordinary straight spokes. Their elastic extension was great enough to render the machine provided with them much more comfortable than one with the ordinary straight spokes.

A spring wheel has the advantage over a spring frame, that it intercepts vibration sooner, so that practically only the wheel rim partakes of the jolting due to the roughness of the road. On the other hand, the springs of a wheel extend and contract once every revolution, and as this cannot be done without the expenditure of energy, a spring wheel must require more power than a rigid wheel to propel it over a good road. The springs of a frame remain quiescent under a steady load while running over a smooth road, only extending or shortening when the wheel passes over a hollow or lump in the road.

In the 'Everett' spring wheel the spokes, instead of being connected directly to the hub, are connected to short spiral springs, thus giving an elastic connection between the hub and the rim, so that the rim may run over an obstacle on the road without communicating much shock to the frame. One objection to a wheel with spring spokes is the want of lateral stiffness of the rim, it being quite easy to deflect the rim sideways by a lateral pressure. The author is inclined to think that this objection may be over-rated, since in a bicycle the pressure on the rim of a wheel must be in, or nearly in, the plane of the wheel. The 'Everett' wheel is satisfactory in this respect. In the 'Persil' spring wheel two rims are used, the springs being introduced between them. The introduction of such a mass of material near the periphery of the wheel will make the bicycle provided with 'Persil' wheels slower in starting than one with ordinary wheels (see sec. 68).

In the 'Deburgo' spring wheel the springs are introduced at the hub, which is much larger than that of an ordinary wheel. Figure 383 shows a section of the 'Deburgo' hub, and figure 384 an end elevation with the outer dust cover removed, so as to show the springs. The outer hub or frame *1*, to which the spokes are attached, is suspended from the inner hub or axle-box, *5*, by spiral springs, *11* and *12*. Frames *2* and *4*, forming rectangular guides at right angles to each other, are fixed respectively to the outer and inner hubs; an intermediate slide, *3*, is formed with corresponding guides, the combination com-

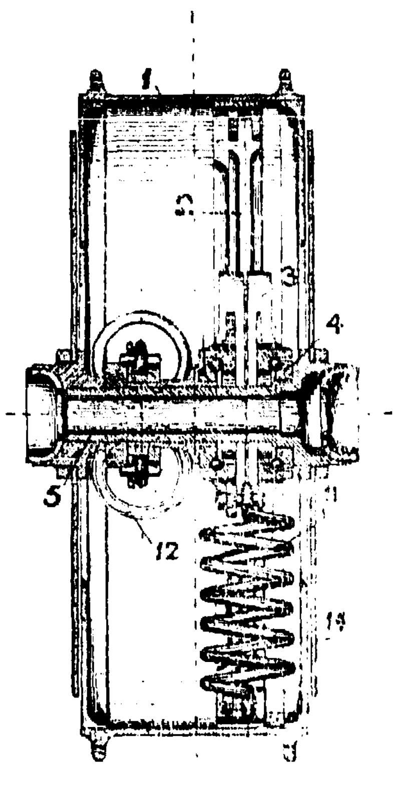


FIG. 383.

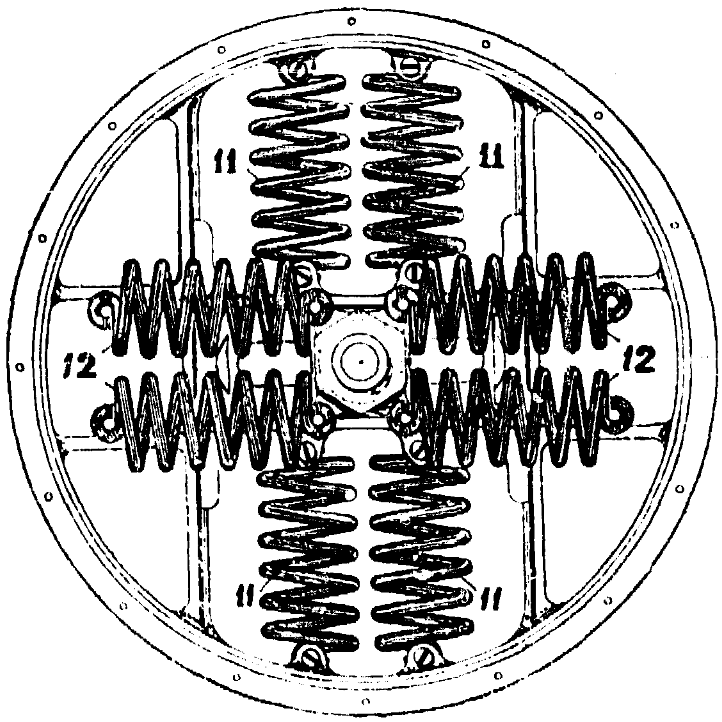


FIG. 384.

pelling the outer to turn with the inner hub, while retaining their axes always parallel to each other, and allowing their respective centres perfect freedom of linear motion. To diminish friction a number of balls are introduced between the slides. Dust-caps, *14*, fixed to the inner hub enclose the springs and guides.

This spring wheel is quite rigid laterally, the only possible relative motion of the outer and inner hubs being at right angles to the direction of their axes.

CHAPTER XXV

BEARINGS

259. **Definition.**—A *bearing* is the surface of contact of two pieces of mechanism having relative motion. In a machine the frame is the structure which supports the moving pieces, which are divided into *primary* and *secondary*, the former being those carried direct by the frame, the latter those carried by other moving pieces. In a more popular sense the bearing is generally spoken of as the portions of the frame and of the moving piece in the immediate neighbourhood of the surface of contact. In this sense the word 'bearing' will be used in this chapter. The bearings of a piece which has a motion of translation in a straight line must have cylindrical or prismatic surfaces, the straight lines of the cylinder or prism being parallel to the direction of motion. The bearings of pieces having rotary motion about a fixed axis must be surfaces of revolution. A part of a mechanism may have a helical motion—that is, a motion of rotation together with a motion of translation in the direction of the axis of rotation; in this case the bearings must be formed to an exact screw.

The three forms of bearing above mentioned correspond to the three lower pairs in kinematics of machinery, viz. the sliding pair, the turning pair, and the screw pair. In each of these three cases the two parts having relative motion may have contact with each other over a surface.

260. **Journal, Pivot, and Collar Bearings.**—Figure 385 shows the simplest form of *journal* bearing for a rotating shaft, the section of the shaft and journal being circular. In this bearing no provision is made to prevent motion of the shaft in the direction of its axis. A bearing in which provision is made

to prevent the longitudinal motion of the shaft is called a *pivot* or *collar* bearing. Figure 386 shows the simplest form of pivot bearing, figure 387 a combined journal and pivot bearing, the end of the shaft being pressed against its bearing by a force in the direction of the axis. Figure 388 shows a simple form of collar bearing in which the same object is attained. A rotating shaft provided with journal bearings may be constrained longitudinally, either by fixing a pivot bearing at each end, or by having a double collar bearing at some point along the shaft. This double collar bearing is usually combined with one of the journals, as at *A* (fig. 389), a collar being formed at each end of the cylindrical bearing. In a long shaft supported by a number of journals it is only necessary to have one double collar bearing; the other bearings should be quite free longitudinally. Thus, in a tricycle axle with four bearings, the best result will be got by having the longitudinal motion of the axle

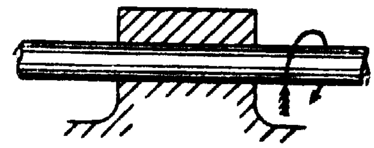


FIG. 385.

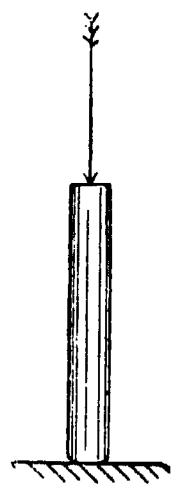


FIG. 386.

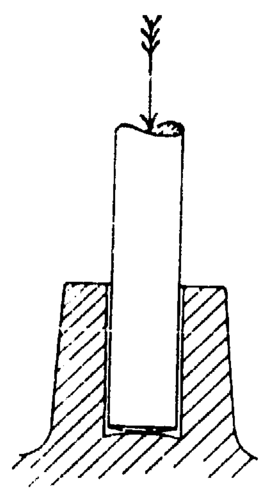


FIG. 387.

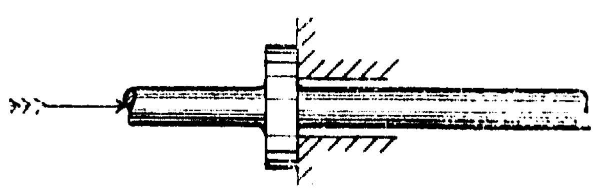


FIG. 388.

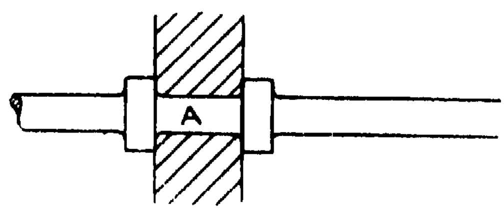


FIG. 389.

controlled at only one of the bearings; if more collars, or their equivalents, are placed on the axle, the only effect is to increase the pressure of the collars on their bearings, and so increase the frictional resistance.

From the point of view of the constraint of the motion it would be quite sufficient for a journal bearing to have contact with the shaft at three points (fig. 390), but as there is usually a considerable pressure on the bearings they would soon be worn. The area of the surfaces of contact should be such that the

pressure per square inch does not exceed a certain limit, depending on the material used and the speed of rubbing.

The bearings of the wheel of an 'Ordinary bicycle were originally made as at *A* (fig. 389), the bearing at each side of the wheel being provided with collars, since the lateral flexibility of the forks was so great that otherwise the bearings would have sprung apart. It was impossible to keep the lubrication of the bearings constantly perfect, and with no film of oil between the surfaces the coefficient of friction rose rapidly and the resistance became serious.

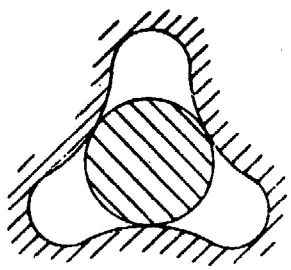


FIG. 390.

Journal Friction.—In a well-designed journal the diameter of the surface of the fixed bearing should be a little greater than that of the rotating shaft (fig. 391). The direction of the motion being then as indicated by the arrow, if the pressure is not too great, the lubricant at *a* is carried by the rotating shaft, and held by capillary attraction between the metal surfaces, so that the shaft is not in actual contact with its bearing, but is separated from it by a thin film of oil. From the experiments carried out by the Institution of Mechanical Engineers it appears that the friction of a perfectly lubricated shaft is very small, the coefficient being in some cases as low as ·001. This compares favourably with the friction of a ball-bearing.

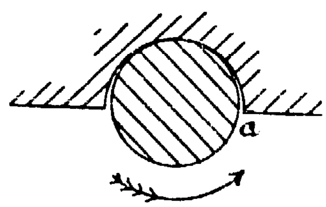


FIG. 391.

Pivot Friction.—With a pivot or collar bearing the case is quite different. The rubbing surface of the shaft is continually in contact with the bearing, and cannot periodically get a fresh supply of oil (as in fig. 391) to keep between the two surfaces. The consequence is that, with the best form of collar bearing, the coefficient of friction is much higher. From the experiments of the Institution of Mechanical Engineers it appears that ·03 to ·06 may be taken as an average value of μ for a well lubricated collar bearing.

261. **Conical Bearings.**—In machinery subjected to much friction and wear, after running some time a shaft will run loose in its bearing. When the slackness exceeds a certain amount the

bearing must be readjusted. One of the simplest means for providing for this adjustment is shown in the *conical bearing* often used for the back wheel of an 'Ordinary' (fig. 392). The hub, H, ran loose on the spindle, S, which was fastened to the fork ends, F_1 and F_2. The surfaces of contact of the hub and spindle were conical, a *loose* cone, C, being screwed on near one

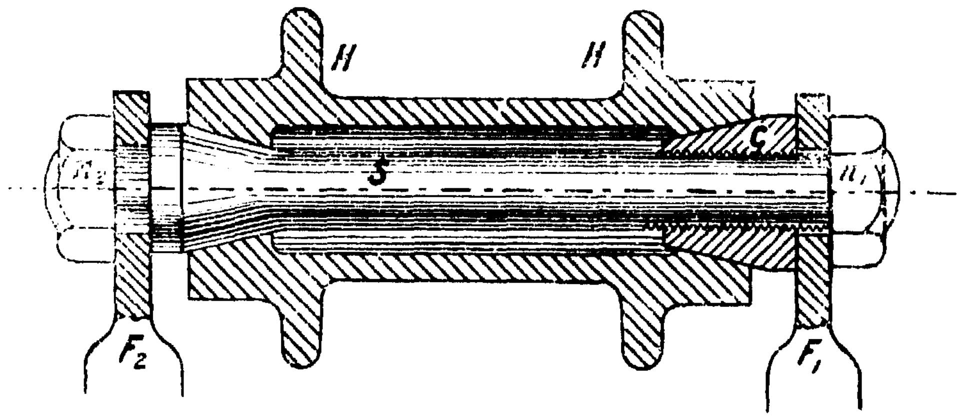


FIG. 392.

end of the spindle. If the bearing had worn loose, the cone C was screwed one or two turns further on the spindle until the shake was taken up. The cone was then locked in position by the nut n_1, which also fastened the end of the spindle to the fork. During this adjustment the other end of the spindle was held rigidly to the fork end F_2, by the nut n_2.

262. **Roller-bearings.**—The first improvement on the plain cylindrical bearing was the *roller-bearing*. Figure 393 is a

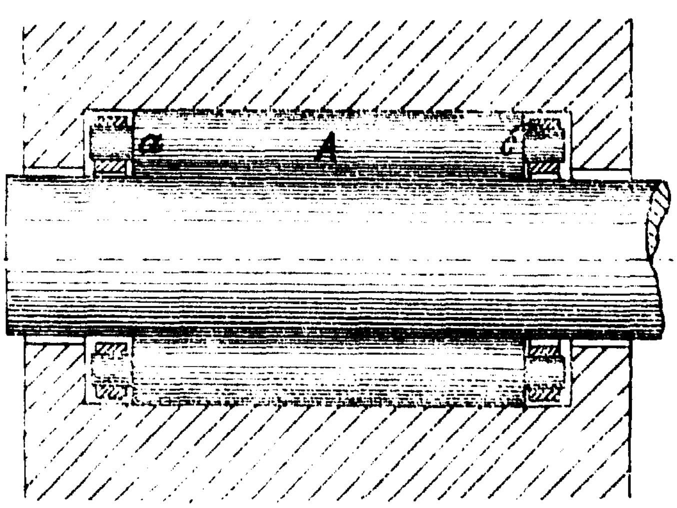


FIG. 393.

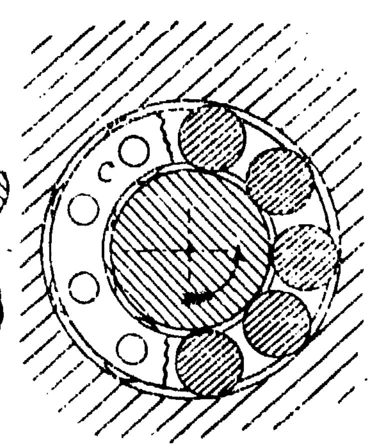


FIG. 394.

longitudinal, and figure 394 an end section of a roller-bearing. In this a number of cylindrical rollers, A, are interposed between

the cylindrical shaft and the bearing-case, the axes of the rollers, *A*, being parallel to that of the shaft. These rollers were sometimes quite loose in the bearing-case, in which case as many rollers as could be placed in position round the shaft were used. More often, however, the ends of the rollers were turned down, forming small cylindrical journals, supported in cages *c*, one at each end of the roller. This cage served the purpose of keeping the distance between the rollers always the same, so that each roller revolved free of the others; whereas, without the cage, two adjacent rollers would often touch, and a rubbing action would occur at the point of contact.

The chief advantage of a roller-bearing over a plain cylindrical bearing is that the lubrication need not be so perfect. While a plain bearing, if allowed to run dry, will very soon get hot; a roller-bearing will run dry with little more friction than when lubricated.

A plain collar bearing must be used in conjunction with a roller-bearing, to prevent the motion of the shaft endways.

263. **Ball-bearings.**—Instead of cylindrical rollers, a number of balls, *B* (fig. 395), might be used. The principal difference in

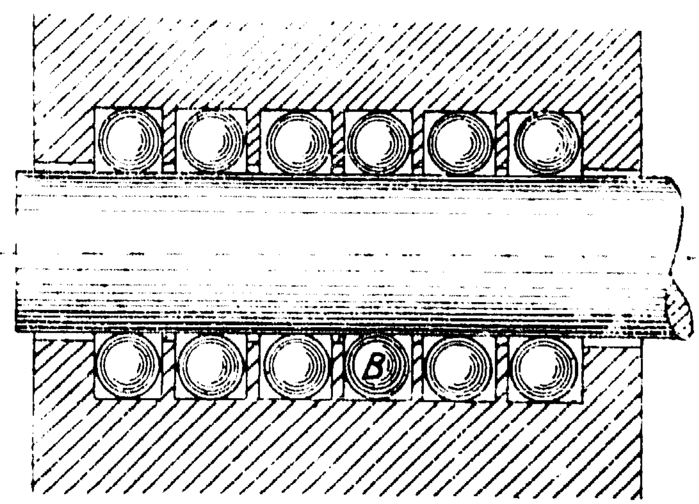


FIG. 395.

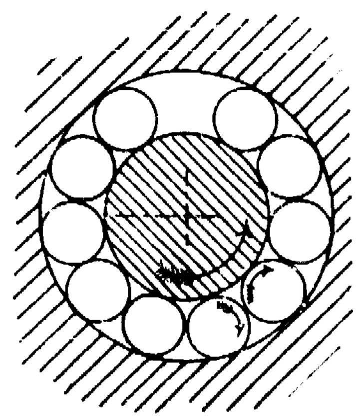


FIG. 396.

this case would be that each ball would have contact with the shaft and the bearing-case at a *point*, while each cylindrical roller had contact along a *line*. As a matter of fact, the surface of contact in the case of the ball-bearing would be a circle of very small diameter (point contact), while in the case of the roller-bearing it would be a very small, narrow rectangle of length equal to that

of the roller (line contact). Other things being equal, the roller-bearing should carry safely a much greater load than the ball-bearing before crushing took place.

The motion of the balls in the bearing shown in figure 395 loaded at right angles to the axis, is one of pure rolling, the axis of rotation of the ball being always parallel to that of the axes of the rolling surfaces of the shaft and bearing-case.

264. **Thrust Bearings with Rollers.**—If a ball- or roller-bearing be required to resist pressure along the shaft, as in figures 386 and 387, the arrangement must be quite different. Two conical surfaces, *a v a* and *b v b*, formed on the frame and the rotating spindle respectively (fig. 397), having a common vertex at *v*, and a common axis coincident with the axis of the spindle, with conical rollers, *a v b*, having the same vertex, *v*, will satisfy the condition of pure rolling. If the axis, *v c*, of the conical roller be supposed fixed, and the spindle be driven, the cone *b v b*

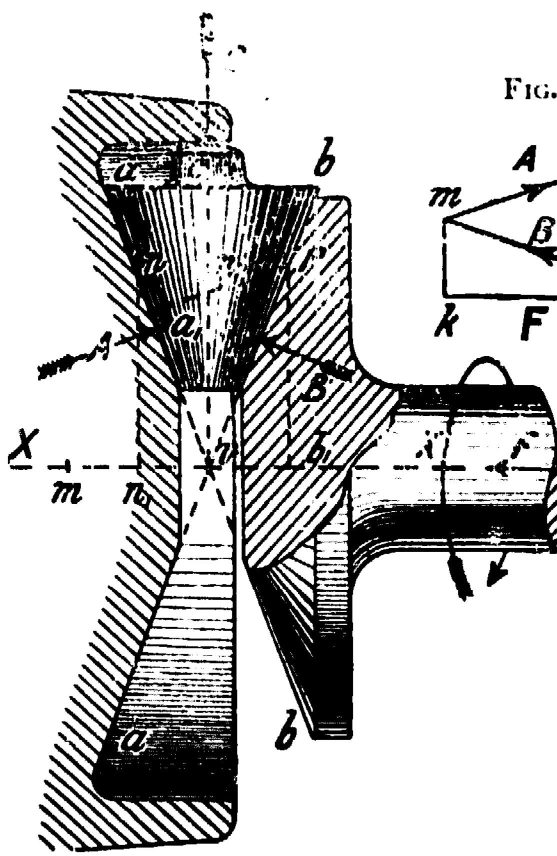
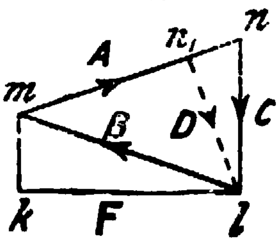


FIG. 397.

FIG. 398.



will drive the roller by friction contact, and it in turn will drive the cone *a v a*. If the cone *a v a* be fixed, and the spindle be driven, the relative motion of the three conical surfaces will remain the same; but in this case the axis of the roller, *v c*, will also rotate about the axis *X X*. With perfectly smooth surfaces, the direction of the pressure is at right angles to the surface of contact, and very nearly so with well lubricated surfaces. On the conical roller, *a v b*, there will therefore be two forces, *A* and *B*, acting at right angles to its sides, *v a* and *v b*, respectively. These have a resultant along the axis *v c*, and unless a third force, *C*, be

applied to the conical roller, it will be forced outwards during the motion.

The magnitudes of the forces A, B, and C can easily be found if the force, F, along the axis is given. In figure 398 draw $l k$ equal to F and parallel to the axis $X X$, draw $l m$ at right angles to $v b$, and $k m$ at right angles to $X X$; $l m$ will give the magnitude of the force B; draw $m n$ and $l n$ respectively at right angles to $v a$ and $a b$, meeting at n; $m n$ and $n l$ will be the magnitudes of the forces A and C respectively.

In figure 397 the conical roller is shown with a prolongation on its axis rubbing against the bearing case, so that its further outward motion is prevented. With this arrangement there will be considerable rubbing friction between the end of the roller and the bearing-case. In Purdon & Walters' thrust bearing for marine engines the resultant outward pressure on the roller is balanced by letting its edge bear against a part of the bearing-case (fig. 399). The generating line, $v a$, of the roller is produced to a point d: $d v_1$ is drawn perpendicular to $v d$, and forms the generating line of a second conical surface coaxial with the first. A small portion on each side of d is the only part of this surface that presses against the bearing-case. The instantaneous axis of rotation being $v d$, there is no rubbing of the roller on the case, but only a relative spinning motion at d. In this case, the force-triangle (fig. 398) will have to be modified by drawing $l n_1$ at right angles to $m n$; $m n_1$ will then be equal to the force A, and $l n_1$ to the pressure D at d.

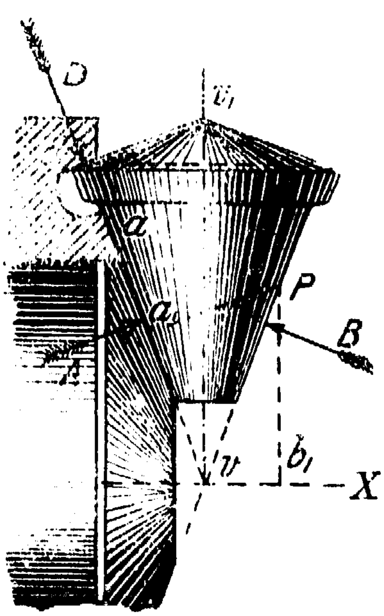


FIG. 399.

Relative Speeds of Roller and Spindle.—Let P (figs. 397 and 399) be any point in the line of contact of the conical roller with the spindle; draw $P a_1$ and $P b_1$ at right angles to $v a$ and $v X$ respectively, and let V be the linear speed of the point P at any instant. Since a_1 is a point on the instantaneous axis of rotation of the roller, and b_1 a point on the fixed axis of rotation of the

spindle, the angular speeds ω_2 and ω_1 of the roller and spindle are respectively

$$\omega_2 = \frac{V}{Pa_1} \text{ and } \omega_1 = \frac{V}{Pb_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Therefore $$\frac{\omega_2}{\omega_1} = \frac{Pb_1}{Pa_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Comparing figures 397, 399, and 398, the triangles $P v b_1$ and $l m k$ are similar; the triangles $P v a_1$ and $l m n_1$ are similar; so also are the four-sided figures $l k m n_1$ and $P b_1 v a_1$. Therefore,

$$\frac{\omega_1}{\omega_2} = \frac{P a_1}{P b_1} = \frac{l n_1}{l k} = \frac{D}{F} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

or, $$F \omega_1 = D \omega_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

That is, if only one roller be used, the angular speeds of the roller and spindle are inversely proportional to the pressures along their instantaneous axes of rotation.

If $v m$ (fig. 397) be set off along the axis of the spindle equal to $P a_1$, and $v n$ along $v a$ equal to $P b_1$, the vectors $v m$ and $v n$ will represent the rotations of the spindle and roller respectively, both in magnitude and direction. $v n$, the rotation of the roller, can be resolved into the rotations $v n_1$ and $v n_2$ about the axes of the shaft and roller respectively. It can easily be shown, from the geometry of the figure, that $v n_1 = \frac{1}{2} v m$; therefore the axis of the roller turns about the axis of the shaft at half the speed of the shaft.

The rotation $v n_1 = \frac{1}{2} \omega_1$, is equivalent to an equal rotation about a parallel axis through c (fig. 397), together with a translation $\frac{1}{2} \omega \times v c$. This translation and rotation constitute a rubbing of the roller on the bearing at c. Thus, finally, the relative motion at c consists of a rubbing with speed $v c \times \frac{1}{2} \omega$ and a spinning with speed $\omega_3 = \overline{v n_2}$.

From figures 397 and 398, $\dfrac{\omega_3}{\omega_1} = \dfrac{v n_2}{2 v n_1} = \dfrac{F}{C}$ (fig. 398).

Therefore, $$F\omega_1 = C\omega_3, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$
and if n rollers be used, with the total thrust W along the shaft,
$$W\omega_1 = nC\omega_3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

If a number of conical rollers are interposed between the two conical surfaces on the shaft and bearing respectively, as in figure 397, the radial thrust, C, on the rollers may be provided for by a steel live-ring against which the ends of the rollers bear. This live-ring will rotate at half the speed of the shaft, and there will be no rubbing of the roller ends relative to it. But it should be noted that the speed of rotation ω_3 of each roller relative to the live-ring will be as a rule greater than the speed of rotation of the shaft, and therefore with a heavy end thrust on the shaft, the risk of abrasion of the outer ends of the rollers will be great. In a

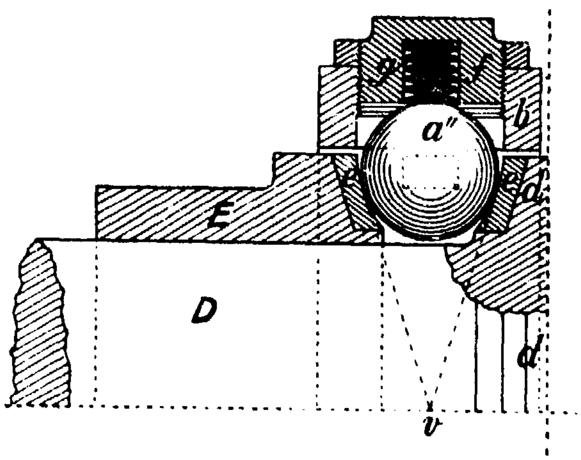


FIG. 400.

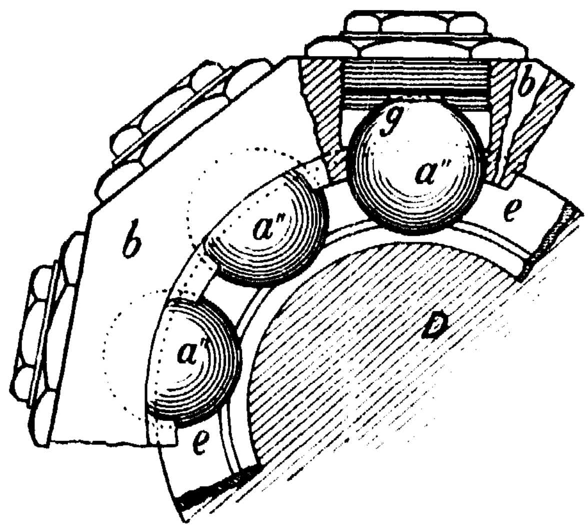


FIG. 401.

thrust bearing for marine engines, designed by the author, a number of lens-shaped steel discs were introduced between the outer end of each roller and the live-ring, so that the average relative spinning motion of two surfaces in contact is made equal to the relative speed between the roller and the live-ring, divided by the number of pairs of surfaces in contact. Figures 400 and 401 show a modification of this design, in which the conical rollers are replaced by balls, a'', rolling between hard steel rings, e, fixed on the shaft and the pedestal respectively. The small portions of these rings and of the balls in contact may be considered as conical surfaces with a common vertex, v. Anti-

friction discs, *g*, are carried in a nut, *f*, which is screwed into and can be locked in position on the live-ring, *b*. This design (figs. 400 and 401) is arranged so that if one ball breaks it can be removed and replaced without disturbing any other part of the bearing. In this thrust-block a plain cylindrical bearing is used to support the shaft.

This bearing may be simplified by the omission of the anti-friction discs, and allowing the balls to run freely in the space

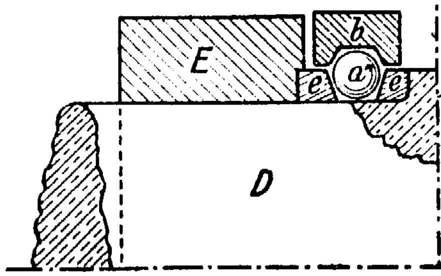


FIG. 402.

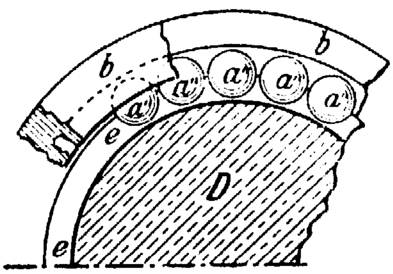


FIG. 403.

enclosed by the two steel rings, *e*, and the live-ring, *b*. Figure 402 is a part longitudinal section of such a simplified thrust bearing, and figure 403 a part cross section.

In a journal bearing the work lost in friction is proportional to the product of the pressure and the speed of rubbing, provided the coefficient of friction remains constant for all loads. In the same way, in a pivot bearing, the work lost in friction—other things being equal—is proportional to the product of the pressure and the angular speed. Equations (4) and (6), therefore, assert that it is impossible by any arrangement of balls or rollers to diminish the friction of a pivot bearing below a certain amount. If a shaft subjected to a longitudinal force can be supported by a plain pivot bearing (fig. 386), the work lost in friction will be a minimum. If, however, the circumstances of the case necessitate a collar bearing (fig. 388), an arrangement of balls or conical rollers may serve to get rid of the friction due to the rubbing of the collar on its bearings. In other words, the effective arm at which the frictional resistance acts may be reduced by a properly designed ball- or roller-bearing to a minimum, so that it may be equivalent to that illustrated in figures 400–1. The pressure on the pivot may sometimes be so great as to make it undesirable to support it by a bearing of the type shown in figure 386 ; the use

of a bearing of either of the types shown in figures 400–1 and 402–3 with a number of balls or conical rollers, is equivalent to the subdivision of the total pressure into as many parts as there are rollers in the bearing.

265. **Adjustable Ball-bearing for Cycles.**—Figure 404 shows diagrammatically one of the forms of ball-bearing used almost universally for cycles. The external load on the bearings of a bicycle or a tricycle is always, with the exception of the ball steering-head, at right-angles to its axis ; any force parallel to the axis being simply due to the reaction of the bearing necessary to keep the spindle in its place. Figure 404 represents the section of the hub of a bicycle wheel ; the spindle, *S S*, is fixed to the

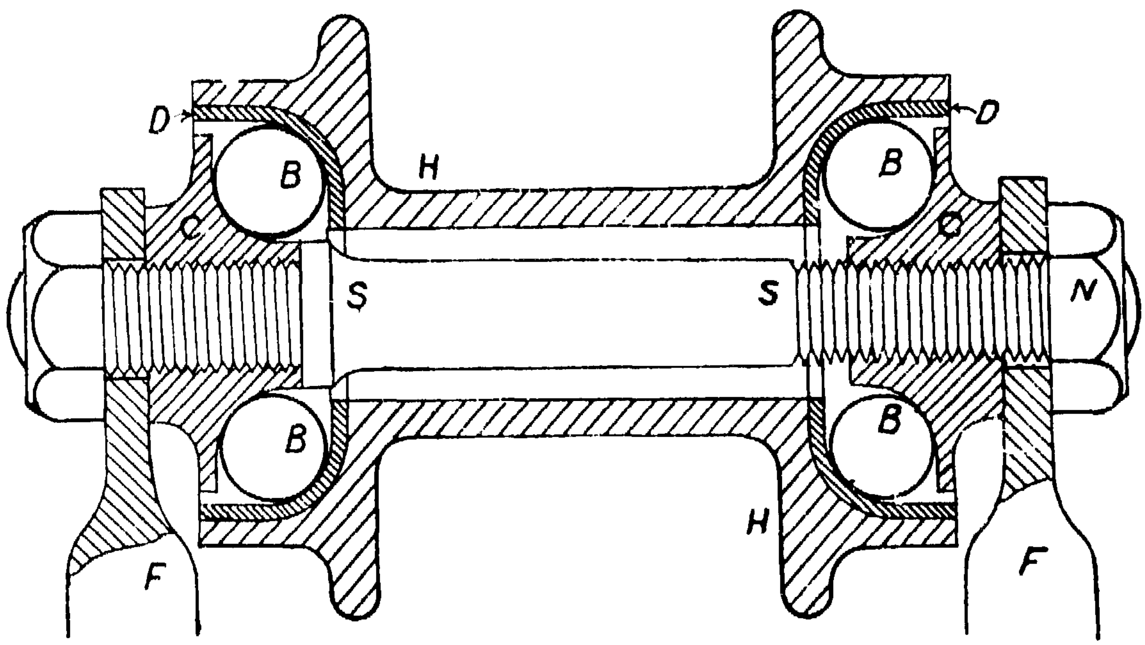


FIG. 404.

fork ; hardened steel 'cones,' *C C*, are screwed on its ends, and hardened steel cups, *D*, are fixed into the ends of the hub, *H*, which is of softer metal. The balls, *B*, run freely between the cone *C* and the cup *D*. One of the cones *C* is screwed up tight against a shoulder of the spindle *S*, the other is screwed up until the wheel runs freely on the spindle without undue shake, it is then locked in position by a lock-nut *N*, which usually also serves to fasten the spindle to the fork end, *F*.

266. **Motion of Ball in Bearing.**—Consider now the equilibrium of the ball *B*. It is acted on by two forces, f_1 and f_2 (fig. 405), the pressure of the wheel and the reaction of the

spindle respectively. Since the ball is in equilibrium, these two forces must be equal and opposite; therefore the points of contact, a and b, of the ball with the cup and cone must be at the extremities of a diameter. During the actual motion in the bicycle the cone C is at rest, the ball B rolls round it, and the cup D rolls on the balls. The relative motion will be the same,

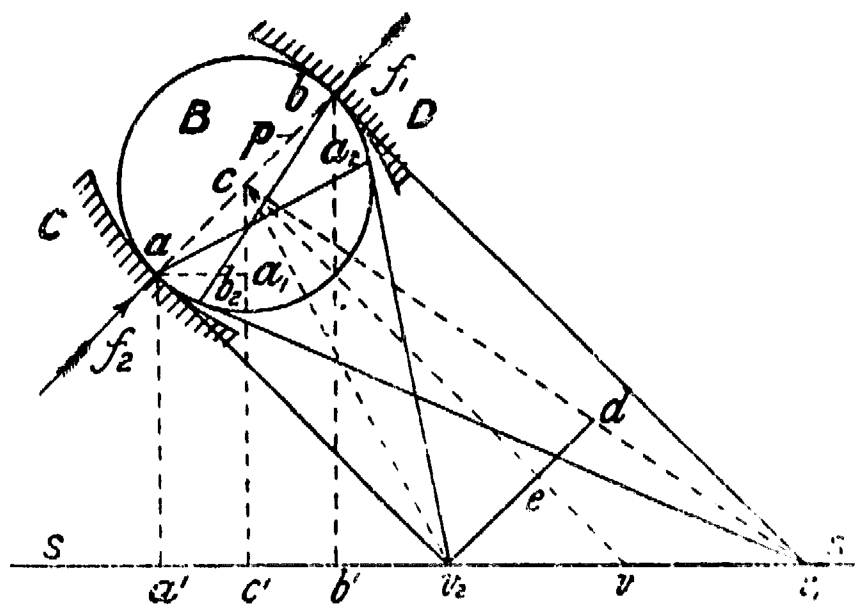


FIG. 405.

however, if a motion of rotation about the axis of the spindle, SS, be impressed on the whole system, equal in amount but opposite in direction to that of the centre of the balls round the axis, SS. The centre c of the ball B may thus be considered to be at rest, the ball to turn about an axis through its centre, the cup D and cone C to rotate in opposite directions about their common axis, SS.

Draw bv_1 at right angles to ab, cutting the spindle SS in v_1 (fig. 405, which is part of fig. 404 to a larger scale); from v_1 draw a tangent $v_1 b_2$ to the circle B, of which ab is the diameter. If the relative motion of the ball and cup at b be one of pure rolling, the portion of the ball in contact at b may be considered as a small piece of a cone bv_1b_2, and the portion of the ball-race at b part of a cone coaxial with SS, both cones having the common vertex v_1. The axis of rotation of the ball will pass through v_1 and the centre c of the ball B.

Now draw av_2 at right angles to ba, cutting SS at v_2. If the

relative motion of the ball and cone at a be one of pure rolling, the portions of the ball and cone surfaces in contact may be considered portions of cones having a common vertex v_2; the axis of rotation of the ball will thus be $c v_2$. But the ball cannot be rotating at the same instant about two separate axes $v_1 c$ and $v_2 c$, so that motions of pure rolling cannot exist at a and b simultaneously. If the surfaces of the cone and the cup be not equally smooth, it is possible that pure rolling may exist at the point of contact of the ball with the rougher surface. Suppose the rougher surface is that of the cup, the axis of rotation of the ball would then be $v_1 c$, and the motion at a would be rolling combined with a spinning about the axis $a c$ at right angles to the surface of contact. Draw $v_2 d$ parallel to $a c$, cutting $v_1 c$ at d. Then, if $c d$ represent the actual angular velocity of the ball about its axis of rotation $v_1 c$, $v_2 d$ will represent the angular velocity of the ball about the axis $a c$; since the rotation cd is the resultant of a rotation $c v_2$ about the axis $c v_2$, and a rotation $v_2 d$ about the axis $c a$: $d c v_2$ is, in fact, the triangle of rotations about the three axes intersecting at c.

If the surfaces of the cone and cup be equally smooth the axis of rotation of the ball will be $c v$, v being somewhere between v_1 and v_2. If the angular speeds of the spinning motions at a and b be equal, $c v$ will bisect $d v_2$. If e be this point of intersection, $e c v_2$ and $e c d$ will be the triangles of rotation at the points a and b respectively.

The above investigation clearly shows that a grinding action is continually going on in all ball-bearings at present used in cycle construction. The grooves formed in the cone and cup after running some time are thus accounted for, while the popular notion that all but rolling friction is eliminated in a well-designed ball-bearing is shown to be erroneous. The effect of this grinding action will depend on the closeness with which the balls fit the cone and cup. If the radii of curvature of the ball, cone, and cup be nearly the same (fig. 406), friction due to the spinning will be great; while, if they

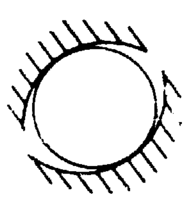


FIG. 406.

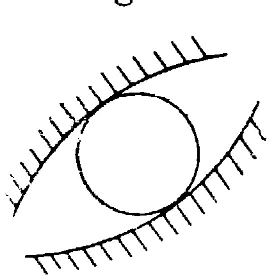


FIG. 407.

are perceptibly different (fig. 407), the friction of the bearing will be much less. On the other hand, a ball in the bearing (fig. 406) will be able to withstand greater pressure than a ball in the bearing (fig. 407), the surface of contact with a given load being so very much less in figure 407 than in figure 406.

267. **Magnitudes of the Rolling and Spinning of the Balls on their Paths.**—From a, c, and b (fig. 405) draw perpendiculars to the axis SS, and let ω be the actual angular speed of the wheel on its spindle, T the sum of the angular speeds of the spinning motions of the ball on its two bearing surfaces, r the radius ca of the ball, and R the radius cc^1 of the circle in which the ball centres run. From a draw aa_1 perpendicular to cc^1. Considering the motion relative to a plane passing through the spindle SS and the line vc—that is, considering the point c to be at rest, as described in section 266—let ω_1 be the angular speed of rotation of the ball about the axis vc, which may be assumed at right angles to ab. The linear speeds of the points a and b of the ball will be $\omega_1 r$. The angular speeds of the spindle and the wheel will be respectively

$$-\frac{\omega_1 r}{aa^1} \text{ and } \frac{\omega_1 r}{bb^1}.$$

But the spindle is actually at rest; so, if the angular speed $\frac{\omega_1 r}{aa^1}$ about the axis SS be now added to the whole system, the actual angular speed of the wheel will be

$$\omega = \left(\frac{1}{aa^1} + \frac{1}{bb^1}\right)\omega_1 r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Denoting the length ca_1 by q,

$$aa^1 = R - q, \text{ and } bb^1 = R + q;$$

equation (7) may therefore be written

$$\omega_1 = \frac{R^2 - q^2}{2Rr}\omega \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

But by section 266

$$T = \frac{v_2 d}{ec}\omega_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Combining (8) and (9)

$$T = \frac{v_2 d}{e c} \cdot \frac{R^2 - q^2}{2 R r} \omega \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (10)$$

An inspection of the diagram (fig. 405) will show that the fractions $\frac{v_2 d}{e c}$ and $\frac{R^2 - q^2}{2 k r}$ are smaller the nearer the diameter $a b$ of contact of the ball with its bearings is to a perpendicular to the spindle $S S$. Also, the distance $e c$ depends on the position of the actual axis of rotation, $c v$, of the ball; but it does not vary greatly, its maximum value being when it coincides with $c v_2$, its minimum when it is perpendicular to $a b$.

The above considerations show that a ball-bearing arranged as in the full lines (fig. 408) will be much better than the one arranged as shown by the dotted lines. The end thrust in bicycle bearings is always small, so that the line of contact $a b$ need not be inclined 45° to the axis, but be placed nearer a perpendicular to the axis.

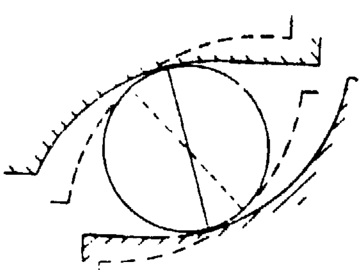


FIG. 408.

The rolling of the balls on the bearings will be much less prejudicial than the spinning; it may be calculated as follows:

The linear speed of the point b of the ball (fig. 405) is $\omega \times b b^1 = (R + q)\, \omega$. The angular speed of rolling of the ball about the axis $a v_2$ is therefore $\frac{(R + q)}{2 r} \omega$. Consider now the outer path D to be fixed, and the inner path C to revolve with the angular speed $-\omega$; the relative motion will be, of course, the same as before The linear speed of the point a of the ball is $\omega \times \overline{a a^1} = (R - q)\, \omega$, and the angular speed of rolling of the ball about the axis $b v_1$ is therefore $\frac{(R - q)}{2 r} \omega$. The sum of the rolling speeds of the ball at a and b is therefore

$$\frac{R}{r} \omega, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (11)$$

a result independent of the angle that the diameter of contact $a b$

makes with the axis of the bearing. The pressure on the ball, however, and therefore also the rolling friction depends on this angle.

Example.—In the bearing of the driving-wheels of a Safety bicycle the balls are $\frac{1}{4}$ in. diameter, the ball circle—that is, the circle in which the centres of the balls lie—is ·8 in. diameter, and the line of contact of the ball is inclined 45°; find the angular speed of the spinning of the balls on their bearing. Figure 409 is the diagram for this case drawn to scale, from which $v_2 d = \cdot 21$ in., $e c = \cdot 44$ in., and $q = \cdot 09$. Substituting these values in (10)

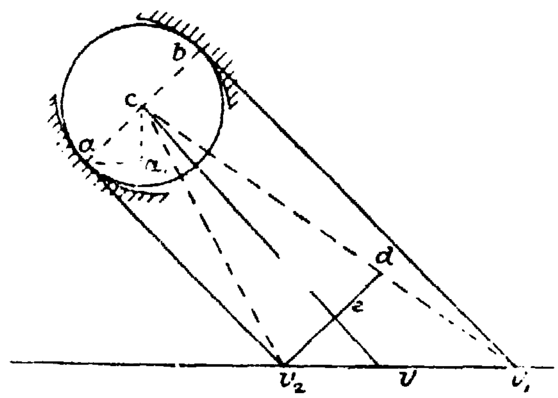


FIG. 409.

$$T = \frac{\cdot 21\,(\cdot 16 - \cdot 0081)}{\cdot 44 \times 2 \times \cdot 4 \times \cdot 125}\,\omega = \cdot 72\,\omega.$$

That is, for every revolution of the hub, the total spinning of each ball relative to the bearings is nearly three-fourths of a revolution.

The pressure on each ball in this case is $\sqrt{2}$ times the *vertical* load on it. Hence the resistance due to spinning friction of the balls will be $\cdot 72\sqrt{2}, = 1 \cdot 018$ times that of a simple pivot-bearing formed by placing a single ball between the end of the pivot and its seat, the total load being the same in each case.

The sum of the speeds of rolling of the ball is, by (11),

$$\frac{\cdot 8}{\cdot 25}\,\omega = 3 \cdot 2\,\omega.$$

268. **Ideal Ball-bearing.**—The external load on the ball-bearing of a cycle is usually at right angles to the axis, but from the arrangement of the bearing (fig. 404) the pressure on the balls has a component parallel to the axis. This component has to be resisted by the bearing acting practically as a collar bearing,

as described in section 260. Thus not only is the actual pressure on the balls increased, but instead of having a motion of pure rolling, a considerable amount of spinning motion under considerable pressure is introduced. The actual force in the direction of the axis necessary to keep the wheel hub in place is very small compared with the total external load; a ball-bearing in which the load is carried by one set of balls, arranged as in figure 395, and the end thrust taken up by another set, might therefore be expected to offer less frictional resistance than those in use at present. Such a bearing is shown in figure 410. The main balls, *B* (fig. 410), transmitting the load from the wheel to the spindle run between coaxial cylindrical surfaces on the spindle and hub respectively; the motion of the balls, *B*, relative to both surfaces, is thus one of pure rolling. The space in which the balls run is a little longer, parallel to the axis of the spindle, than their diameter, so that they do not bear sideways. The wheel is kept in position along the spindle by a set of balls, *b*, running between two conical surfaces on the spindle and hub respectively, having a common vertex, and kept radially in place by a live-ring, *r*. One of these cones is fixed to the spindle, the other forms part of the main ball cup. This bearing is therefore a combination of the ball-bearing (fig. 395) and the thrust bearing (fig. 402–3). The motion of the main balls, *B*, being pure rolling, the necessity of providing means of adjustment will not be so great as with the usual form; in fact, the bearing being properly made by the manufacturer may be sent out without adjustment. A play of a hundredth part of an inch might be allowed in the two main rows of balls, *B*, and a longitudinal play of one-twentieth of an inch for the secondary rows, *b*. If the main row of balls ultimately run loose, a new hard steel ring, *R*, can be easily slipped on the spindle.

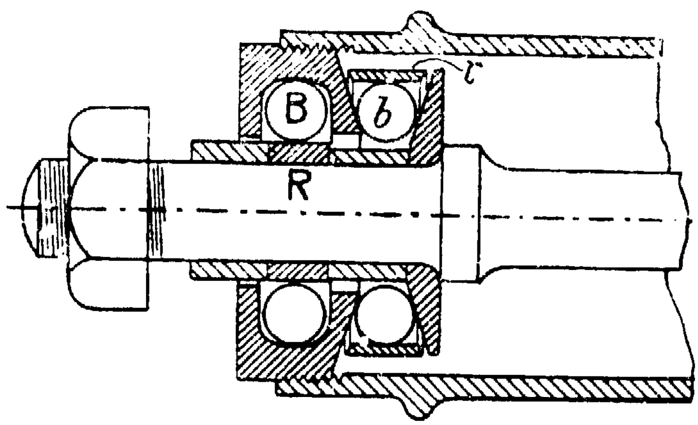


FIG. 410.

If adjustments for wear are required in this type of bearing, they can be provided by making the hard steel ball ring, *R*, slightly tapered (fig. 411), and screwing it on the spindle. It would be locked in position by the nut fixing the spindle to the frame. There would be an adjustment at each end.

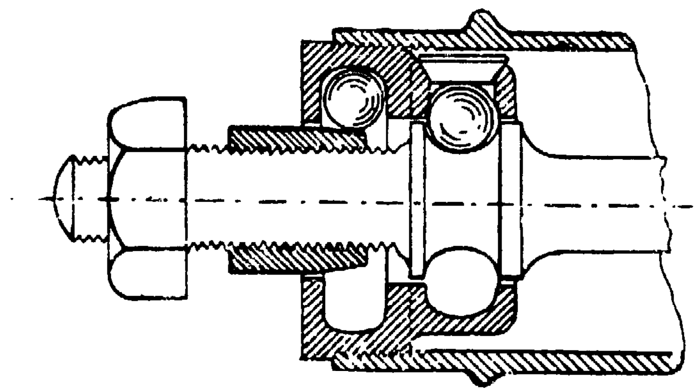


FIG. 411.

These bearings may be somewhat simplified in construction, though the frictional resistance under an end thrust will be theoretically increased, by omitting the live-ring confining the secondary balls, and merging it in either the cup or the conical disc (fig. 412). If this be done a single ball will probably be sufficient for each row of secondary balls, *b*. If a double collar be formed near one end of the spindle, one row of secondary balls, *b*, would be sufficient for the longitudinal constraint. They could be put in place through a hole in the ball cup (fig. 411), or by screwing an inner ring on the cup

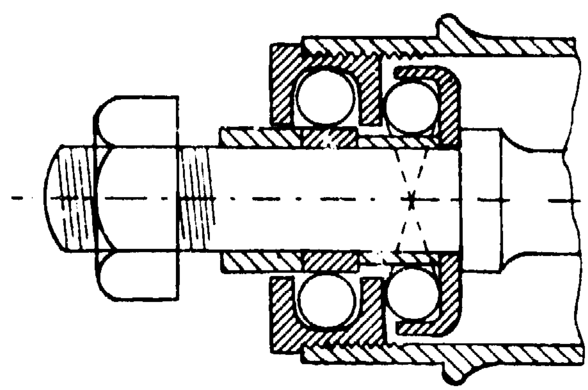


FIG. 412.

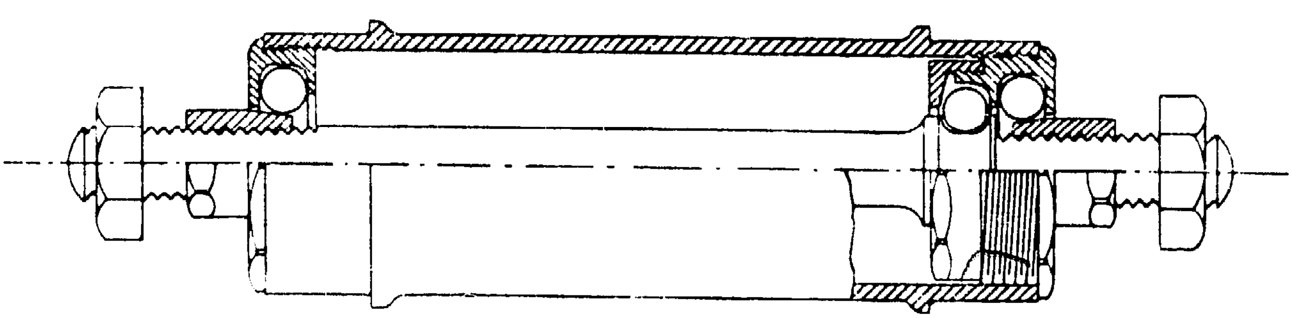


FIG. 413.

(fig. 413). The other end of the bearing will have only the main row of balls.

269. **Mutual Rubbing of Balls in the Bearing.**—Figure 414 may be taken to represent a section of a ball-bearing by a plane at right angles to the axis, the central spindle being fixed and the outer case revolving in the direction of the arrow a. The balls will therefore roll on the fixed spindle in the direction indicated. If two adjacent balls, B_1 and B_2, touch each other there will be rubbing at the point of contact, and of course the friction resistance of the bearing will be increased. Now, in a ball-bearing properly adjusted the adjusting cone is not screwed up quite tight, but is left in such a position that the balls are not all held at the same moment between the cones and cups; in other words, there is a little play left in the bearing. Figure 414 shows such a bearing sustaining a vertical load, as in the case of the steering-wheel of a bicycle, with the play greatly exaggerated for the sake of clearness of illustration. The cone on the wheel spindle will rest on the balls near the lowest part of the bearing, and the balls at the top part of the bearing will rest on the cone, but be clear of the cup of the wheel. Thus, a ball in its course round the bearing will only be *pressed* between the two surfaces while in contact at any point of an arc, c_1 c_2, and will run loose the rest of the revolution. The balls should never be jammed tightly round the bearing, or the mutual rubbing friction will be abnormally great. The ascending balls will all be in contact, the mutual pressure being due merely to their own weight. A ball, B_3, having reached the top of the bearing will roll slightly forward and downward, until stopped by the ball in front of it, B_4. The descending balls will all be in contact, the mutual pressure being again due to their own weight. On coming into action at the arc c_1 c_2, the pressure on the balls tends to flatten them slightly in the direction of the pressure, and to extend them slightly in all directions at right angles. The mutual pressure between the balls may thus be slightly increased, but it is probable that it cannot be much greater than that due to the weight of the descending balls. As this only amounts to a very small fraction of an ounce, in com-

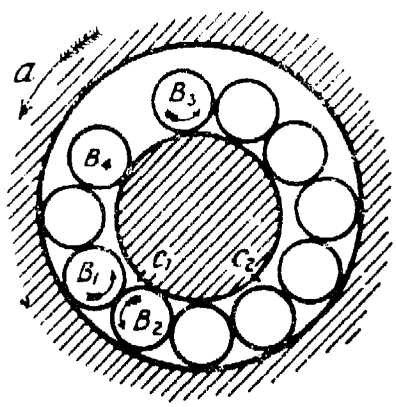


FIG. 414.

parison with the spinning friction above described under a total load of perhaps 100 lbs., the friction due to the balls rubbing on each other is probably negligibly small.

Figure 414 represents the actions in the bearings of non-driving wheels of bicycles and tricycles, and in the driving-wheels of chain-driven Safety bicycles ; also, supposing the outer case fixed and the inner spindle to revolve, it represents the action in the crank-bracket of a rear-driving Safety.

In the bearings of the front wheel of an 'Ordinary,' or the front-driving Safety, the action is different, and is represented in figure 415. In these cases the balls near the upper part of the bearing transmit the pressure, the lower balls being idle. The motion being in the direction shown by the arrow a, the ball B_1 is just about to roll out of the arc of action, and will drop on the top of the ball B_2. The ball B_3, ascending upwards, will move into the arc of action c_1 c_2, and will be carried round, while the ball behind it, B_4, will lag slightly behind. In this way, it is possible that there may be no actual contact between the balls transmitting the pressure.

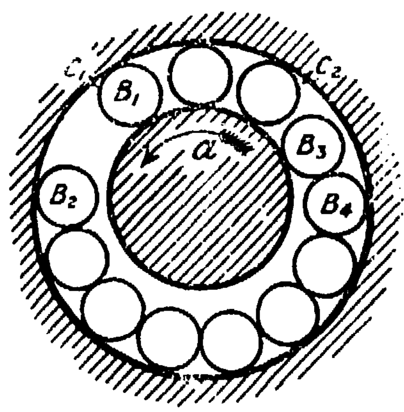


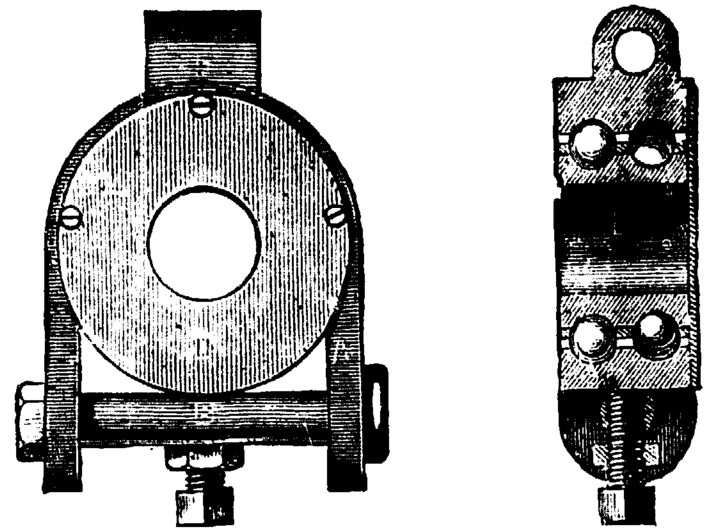
FIG. 415.

It would be interesting to experiment on the coefficient of friction of the same ball-bearing under the two different conditions illustrated in figures 414 and 415. In some of the earlier ball-bearings the balls were placed in cages, so as to prevent their mutual rubbing. Figures 416 and 417 show the 'Premier' bearing with ball-cage. It does not appear that the rubbing of the balls on the sides of the cage is less prejudicial than their mutual rubbing ; and as, with a cage, a less number of balls could be put into a bearing, cages were soon abandoned.

Effect of Variation in Size of Balls.—If one ball be slightly larger than the others used in the bearing, it will, of course, be subjected to a greater pressure than the others ; in fact, the whole load of the bearings may at times be transmitted by it, and there will be a probability of it breaking and consequent damage to the surface of the cone and cup. Let V be the linear speed of the

point of the cup in contact with the ball (fig. 414), R the radius of the ball centre, and r the radius of the ball; the linear speed of the ball centre is $\frac{V}{2}$, and its angular speed round the axis of

FIG. 416.



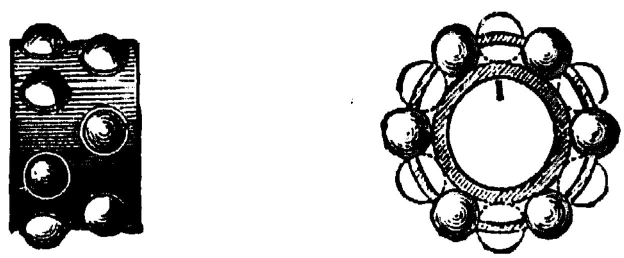


FIG. 417.

the spindle is $\frac{V}{2R}$. The radius R is the sum of the radii of a ball and of the circle of contact with the cone; consequently the angular speed round the centre of the spindle of a ball slightly larger than the others will be less than that of the others, the large ball will tend to lag behind and press against the following ball.

If P be the bearing pressure on the large ball, the mutual pressure, F, between it and the following ball may amount to μP, and the frictional resistance of the bearing will be largely increased.

The mutual rubbing of the balls may be entirely eliminated by having the balls which transmit the pressure alternating with others slightly smaller in diameter. The latter will be subjected only to the mutual pressure between them and the main balls, and will rotate in the opposite direction. They may rub on the

bearing-case or spindle, but, since the pressure at these points approaches zero, there will be very little resistance. This device may be used satisfactorily in a ball-thrust bearing, but in a bicycle ball-bearing the number of balls in action at any moment may be too small to permit of this.

270. **The Meneely Tubular Bearing.**—In the Meneely tubular bearing, made by Messrs. Siemens Brothers (fig. 418), the mutual rubbing of the rollers is entirely eliminated by an ingenious arrangement. "The bearing is composed of steel tubes, uniform in section, which are grouped closely, although not in contact with each other, around and in alignment with the

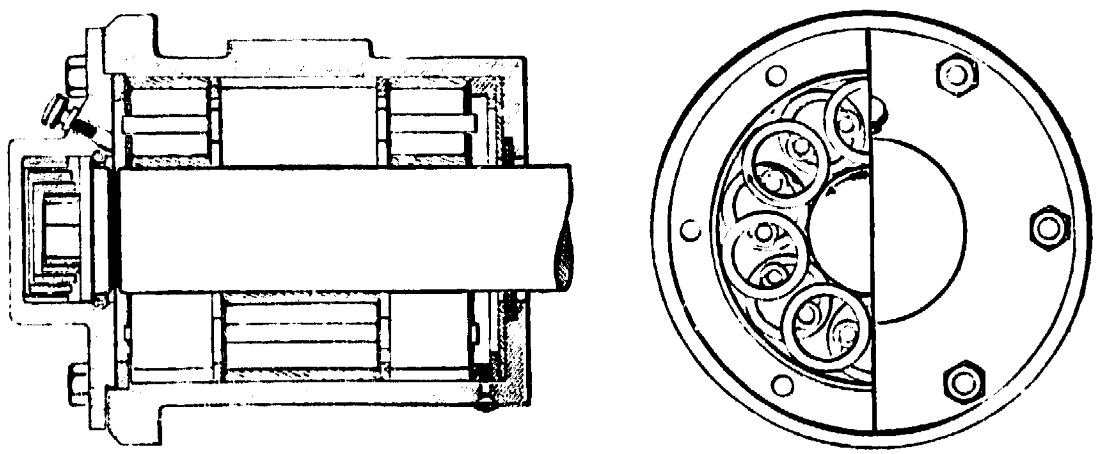


FIG. 418.

journal: these rollers are enclosed within a steel-lined cylindrical housing. They are arranged in three series, the centre series being double the length of the outer series. Each short tube is in axial alignment with the corresponding tube of the opposite end series, while exactly intermediate to these end lines are arranged the axes of the centre series, thus making the lines of bearing equal. Each end tube overlaps two centre tubes, as shown in figure 418. To keep the long and short tubes in proper relative position, there are threaded through their insides round steel rods. These rods both lock the rollers together and hold them apart in their proper relative position, collars on the rods also serving to aid in maintaining the endwise positions. These connecting-rods share in the general motion, rolling without friction in contact with the tubes. They intermesh the long and short tubes, and keep them rigidly in line with the axis." For a

journal 3 in. diameter, the external and internal diameters of the rollers are 2 in. and $1\frac{1}{2}$ in. respectively.

271. **Ball-bearing for Tricycle Axle.**—Figure 419 represents a form of ball-bearing often used for supporting a rotating axle, as the front axle of an 'Ordinary,' tricycle axles, &c. This bearing supports the load at right angles to the axle and at the same time resists end-way motion. A ball has contact with the ball-races at four points, *a*, *b*, *c*, *d*, which for the best arrangement should be in pairs parallel to the axis; the motion of the ball will then be one of rotation, the instantaneous axis being *c d*, its line of contact with the bearing case. The motion of the ball relative to any point of the surface it touches will, however, be one of rolling combined with spinning about an axis perpendicular to the surface of contact.

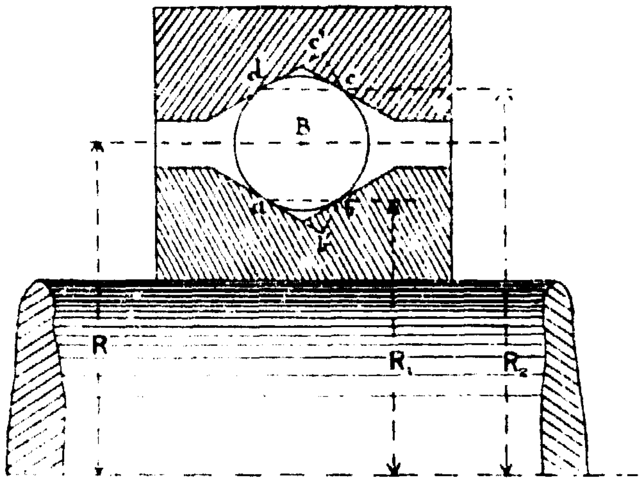
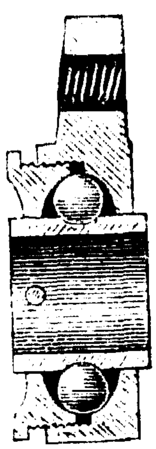


FIG. 419.

Figure 420 shows this form as made adjustable by Mr. W. Bown. The outer ball-cup is screwed into the bearing case, and when properly adjusted is fixed in position by a plate and set screw. If this bearing be attached to the frame or fork by a bolt having its axis at right angles to the rotating spindle, it will automatically adjust itself to any deflection of the frame or spindle; the axes of the spindle and bearing case always remaining coincident.



B

FIG. 420.

Let ω be the angular speed of the axle, r the radius of the ball, R_1 R, and R_2 the distances of the points *a*, *B*, and *d* from the axis *S S*. The linear speed of the point *a* common to the ball and the axle will be ωR_1. The angular speed of the ball about its instantaneous axis of rotation *d c* will be

$$-\frac{\omega R_1}{a d} = -\frac{\omega R_1}{R_2 - R_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The relative angular speed of the ball and axle about their instantaneous axis $a\,b$ will be

$$\omega + \frac{\omega R_1}{R_2 - R_1} = \frac{\omega R_2}{R_2 - R_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Draw $c\,c^1$ at right angles to the tangent to the ball at d; then at the point d the actual rotation of the ball about the axis $c\,d$ can be resolved into a rolling about the axis $d\,c^1$ and a spinning about an axis $d\,B$ at right angles; $d\,c\,c^1$ will be the triangle of rotations at the point d. If the angular speed of spinning of the ball at d is T_d, we have

$$-\frac{T_d}{\dfrac{\omega R_1}{R_2 - R_1}} = \frac{c\,c^1}{c\,d}, \quad i.e., \quad T_d = -\frac{c\,c^1}{c\,d} \cdot \frac{R_1}{(R_2 - R_1)}\omega \quad . \quad (14)$$

Draw $b\,b^1$ perpendicular to the tangent at a; then, in the same way, it may be shown that the angular speed of the relative spinning at the point a is

$$T_a = \frac{b\,b^1}{b\,a} \cdot \frac{R_2}{(R_2 - R_1)}\omega \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

From (14) and (15) the speeds of spinning at a and d are inversely proportional to the radii; the circumferences of the bearings at a and b are also proportional to the radii. If the wear of the bearing be proportional to the relative spinning speed of the ball, and inversely proportional to the circumference—both of which assumptions seem reasonable—the wear of the inner and outer cases at a and d will be inversely proportional to the squares of their radii. If the bearing surfaces at a, b, c, and d are all equally inclined to the axis, $\frac{c\,c^1}{c\,d} = \frac{b\,b^1}{b\,a}$; then adding (14) and (15), the sum of the angular speeds of spinning at a, b, c, and d will be

$$T = 2\frac{c\,c^1}{c\,d} \cdot \frac{R_2 + R_1}{R_2 - R_1}\omega$$

$$= 4\frac{c\,c^1}{c\,d} \cdot \frac{R}{a\,d}\omega \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

If θ be the angle that the tangent $d\,c^1$ makes with the axis of the bearing, $\frac{c\,c^1}{c\,d} = \sin\theta$, $a\,d = 2\,r\,\cos\theta$, and (16) may be written,

$$T = \frac{2\,R\tan\theta}{r}\,\omega \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Equation (17), therefore, shows that the spinning motion in this form of bearing is proportional to the radius of the ball circle, inversely proportional to the radius of the ball, and directly proportional to the tangent of the angle the bearing surfaces make with the axis.

Example.—Let the four bearing surfaces be each inclined 45° to the axis; then $\tan\theta = 1$, and (17) becomes

$$T = \frac{2\,R}{r}\,\omega \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

If the diameter of the ball is $\frac{1}{4}''$, $r = \frac{1}{8}$, and if R is $\frac{1}{2}''$; substituting in (18),

$$T = 8\omega.$$

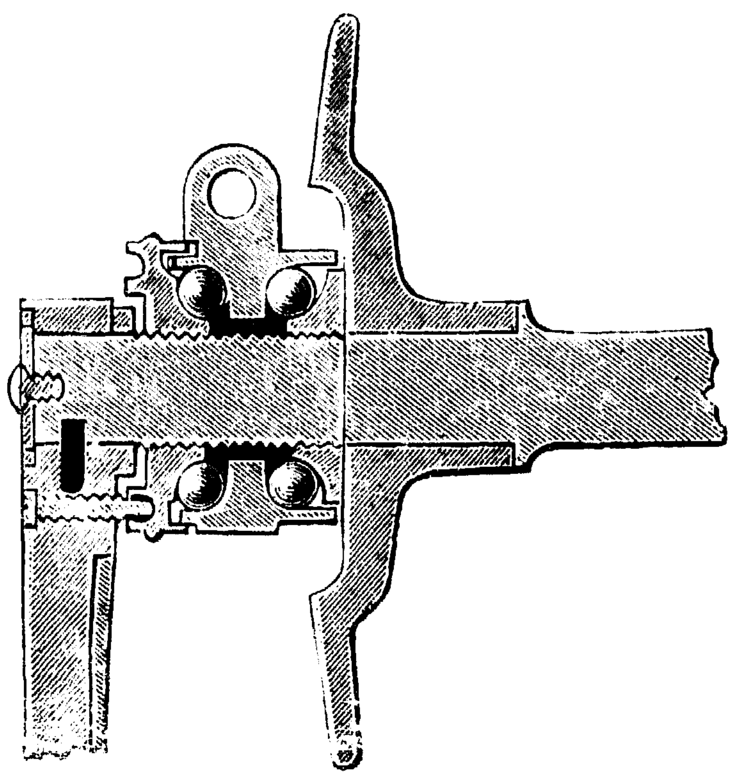


FIG. 421.

This gives the startling result that for every turn of the axle each ball has a total spinning motion of eight turns relative to the surfaces it touches. This form of bearing, therefore, is much inferior to the *double* ball-bearing, which was much used for the front wheels of 'Ordinaries.' Figure 421 is a sectional view of a double ball-bearing as used by Messrs. Singer & Co. The motion of the balls in this bearing is the same as that analysed in section 266.

272. **Ordinary Ball Thrust Bearing.**—Figure 422 is a section of a form of ball thrust bearing which is sometimes used in light drilling and milling machines. The lower row in the ball-head of a cycle also forms such a bearing.

The arrangement of the ball and its grooves, shown in figure 422, is almost as bad as it could possibly be. Let *a*, *b*, *c*,

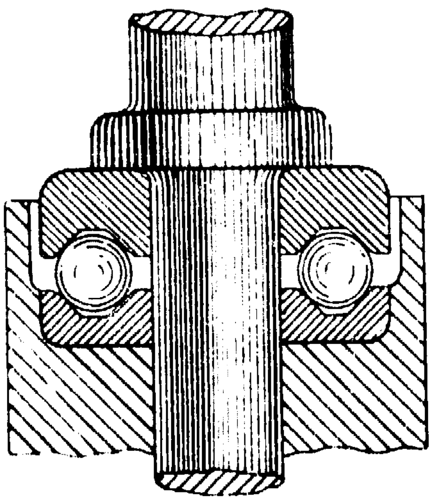


FIG. 422.

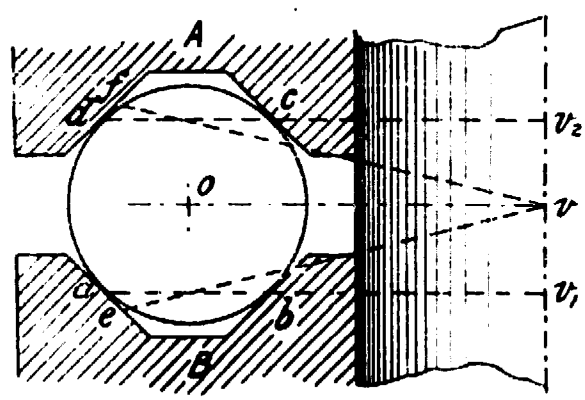


FIG. 423.

and *d* (fig. 423) be the points of contact of the ball with the sides of the groove, and *o* the centre of the ball. If no rubbing takes place at the points *a* and *b*, the instantaneous axis of rotation of the ball relative to the groove *B* must be the line *a b*; that is, the motion of the ball is the same as that of a cone, with vertex v_1 and semi-angle $o\, v_1\, a$, rolling on the disc of which the line $a\, v_1$ is a section. Suppose now that there is no rubbing at the point *d*, and let ω be the angular speed of the spindle *A*. Drop a perpendicular $d\, v_2$ on to the axis. Then V_d, the linear speed of the point *D*, will be

$$\omega \times \overline{v_2\, d},$$

and V_c, the linear speed of the point *c* of the spindle, will be

$$\omega \times \overline{v_2\, c}.$$

The angular speed of the ball is

$$\frac{V_d}{\overline{d\, a}} = \omega \times \frac{\overline{v_2\, d}}{\overline{d\, a}}.$$

The linear speed of the point c on the ball must be equal to the speed of the point d on the ball, since these two points are at the same distance from the instantaneous axis of rotation $a\,v_1$. Therefore the speed of rubbing at the point c is

$$\omega \times (\overline{v_2\,d} - \overline{v_2\,c})$$

$$= \omega \times \overline{c\,d}.$$

If the grooves are equally smooth it seems probable that the actual vertex, v, of the rolling cone will be about midway between v_1 and v_2, and the rolling cone, equivalent to the ball, will be $c\,v\,f$; the points a and d will lie inside this cone, the points b and c outside, and the rubbing will be equally distributed between the points a, b, c, and d.

In comparison with the rubbing, the rolling and spinning frictions will be small. A much better arrangement would be to have only one groove, the other ball-race being a flat disc.

273. **Dust-proof Bearings.**—If the ball-bearing (fig. 404) be examined it will be noticed that there is a small space left between the fixed cone C, and the cup D, fastened to the rotating hub. This is an essential condition to be attended to in the design of ball-bearings. If actual contact took place between the cone and the cup, the rubbing friction introduced would require a greater expenditure of power on the part of the rider. Now, for a ball-bearing to work satisfactorily, the adjusting cone should not be screwed up quite tight, but a perceptible play should be left between the hub and the spindle; the clearance between the cup C and cone D should therefore be a little greater than this.

In running along dusty roads it is possible that some may enter through this space, and get ground up amongst the balls. In so-called dust-proof bearings, efforts are made to keep this opening down to a minimum, but no ball-bearing can be absolutely dust-proof unless there is actual rubbing contact between the rotating hub and a washer, or its equivalent, fastened to the spindle. Approximately dust-proof bearings can be made by arranging that there shall be no corners in which dust may easily find a lodgment. Again, it will be noticed that the diameter of the annular opening for the ingress of dust is smaller in the bear-

ing figure 413 than in the bearing figure 404; the former bearing should, therefore, be more nearly dust-proof than the latter.

The small back wheels of 'Ordinaries' often gave trouble from dust getting into the bearings, such dust coming, not only from the road direct, but also being thrown off from the driving-wheel. When a bearing has to run in a very dusty position a thin washer of leather may be fixed to the spindle and press lightly on the rotating hub, or *vice versâ*. The frictional resistance thus introduced is very small, and does not increase with an increase of load on the bearing.

274. **Oil-retaining Bearings.**—Any oil supplied through a hole at the middle of the hub in the bearing shown in figure 404 will sooner or later get to the balls, and then ooze out between the cup and cone. In the bearing shown in figure 413, on the other hand, the diameter of the opening between the spindle and hub being much less than the diameter of the outer ball-race, oil will be retained, and each ball at the lowest part of its course be immersed in the lubricant.

Figure 380 is a driving hub and spindle, with oil-retaining bearings, made by Messrs. W. Newton & Co., Newcastle-on-Tyne. Figure 381 is a hub, also with oil-bath lubrication, by the Centaur Cycle Company.

275. **Crushing Pressure on Balls.**—In a row of eight or nine balls, all exactly of the same diameter and perfectly spherical, running between properly formed races, it seems probable that the load will be distributed over two or three balls. If one ball is a trifle larger than the others in the bearing, it will have, at intervals, to sustain all the weight. In a ball thrust bearing with balls of uniform size, the total load is distributed amongst all the balls. The following table of crushing loads on steel balls is given by the Auto-Machinery Company (Limited), Coventry, from which it would appear that if P be the crushing load in lbs., and d the diameter of the ball in inches,

$$P = 82400\, d^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

TABLE XIII. WEIGHTS, APPROXIMATE CRUSHING LOADS, AND SAFE WORKING LOADS OF DIAMOND CAST STEEL BALLS.

Diameter of ball	Weight per gross	Crushing load	Working load
in.	lbs.	lbs.	lbs.
$\frac{1}{8}$	·0415	1,288	160
$\frac{3}{16}$	·1401	2,900	360
$\frac{1}{4}$	·3322	5,150	640
$\frac{5}{16}$	·6488	8,050	1,000
$\frac{3}{8}$	1·1213	11,600	1,450
$\frac{1}{2}$	2·6576	20,600	2,570

276. **Wear of Ball-bearings.**—It is found that the races in ball-bearings are grooved after being some time in use. This grooving may be due partly to an actual removal of material owing to the grinding motion of the balls, and partly to the balls gradually pressing into the surfaces, the balls possibly being slightly harder than the cups and cones.

Professor Boys has found that the wear of balls in a bearing is practically negligible ('Proc. Inst. Mech. Eng.,' 1885, p. 510).

277. **Spherical Ball-races.**—If by any accident the central spindle in a ball-bearing gets bent, the axes of the two ball-races will not coincide, and the bearing may work badly. Messrs. Fichtel & Sachs, Schweinfurt, Germany, get over this difficulty

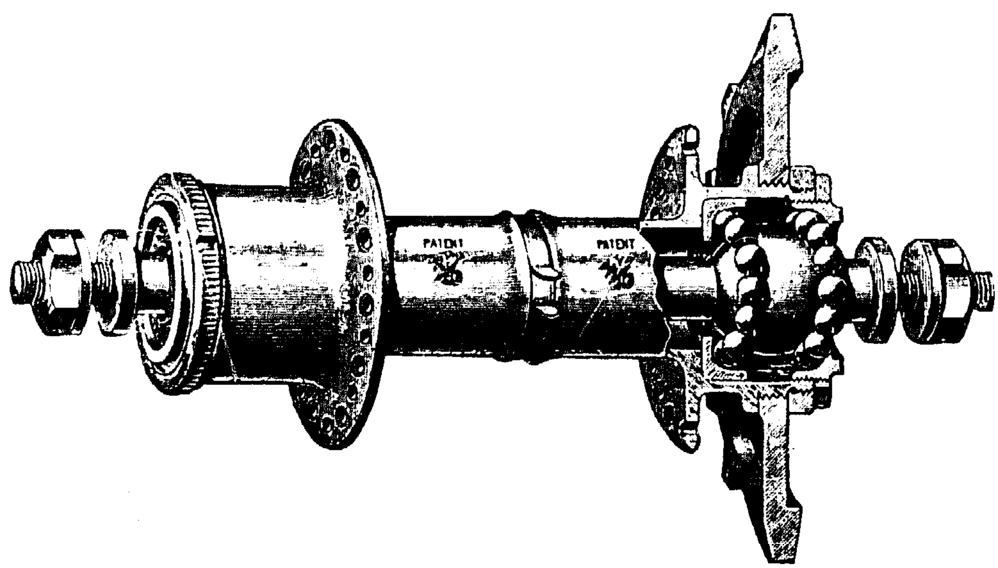


FIG. 424.

by making the inner ball-race spherical (fig. 424), so that however the spindle be bent the ball-race surface will remain unaltered.

278. **Universal Ball-bearing.** — Figure 425 shows a ball-bearing designed by the author, in which either the spindle or the hub may be considerably bent without affecting its smooth running. The cup and cone between which the balls run, instead of being rigidly fixed to the hub and spindle respectively, rest on concentric spherical surfaces. One of the spindle spherical surfaces is made on the adjusting nut. This bearing, automatically adjusting itself, requires no care to be taken in putting it together. The working parts, being loose, can be renewed, if the necessity arises, by an unskilled person.

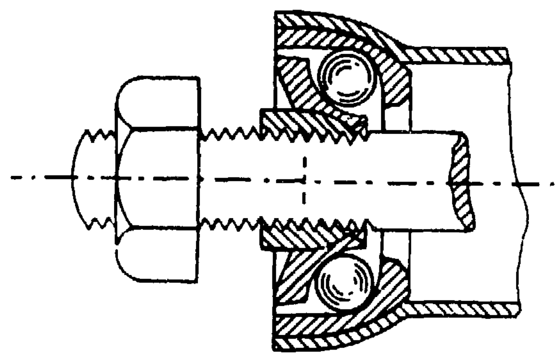


FIG. 425.

In the case of a bicycle falling, the pedal-pin runs a great chance of being bent, a bearing like either of the two above described seems therefore desirable for, and specially applicable to, pedals.

CHAPTER XXVI

CHAINS AND CHAIN GEARING

279. **Transmission of Power by Flexible Bands.**—A flexible steel band passing over two pulleys was used in the 'Otto' dicycle to transmit power from the crank-axle to the driving-wheels. The effort transmitted is the difference of the tensions of the tight and slack sides of the band; the maximum effort that can be transmitted is therefore dependent on the initial tightness. Like belts or smooth bands, chains are flexible transmitters. If the speed of the flexible transmitter be low, the tension necessary to transmit a certain amount of power is relatively high. In such cases the available friction of a belt on a smooth pulley is too low, and gearing chains must be used. Projecting teeth are formed on the drums or wheels, and fit into corresponding recesses in the links of the chain.

A chain has the advantage over a band, that there is, or should be, no tension on its slack side, so that the total pressure on the bearing due to the power transmitted is just equal to the tension on the driving side.

For chain gearing to work satisfactorily, the pitch of the chain should be equal to that of the teeth of the chain-wheels over which it runs. Unfortunately, gearing chains subjected to hard work gradually stretch, and when the stretching has exceeded a certain amount they work very badly.

Gear.—The total effect of the gearing of a cycle is usually expressed by giving the diameter of the driving-wheel of an 'Ordinary' which would be propelled the same distance per turn of the pedals. Thus, if a chain-driven Safety has a 28-in. driving-wheel which makes two revolutions to one of the crank-axle, the

machine is said to be geared to 56 in. Let N_1 and N_2 be the numbers of teeth on the chain-wheels on driving-wheel hub and crank-axle respectively, and d the diameter of the driving-wheel in inches, then the machine is geared to

$$\frac{N_2}{N_1} d \text{ inches} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

The distance travelled by the machine and rider per turn of the crank-axle is of course

$$\frac{N_2}{N_1} \pi d \text{ inches} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

The following table of gearing may be found useful for reference :

TABLE XIV.—CHAIN GEARING.

Gear to which Cycle is Speeded.

Number of teeth on		Diameter of driving wheel											
Crank-axle	Hub	22	24	26	28	30	32	34	36	38	40	42	44
16	7	$50\frac{2}{7}$	$54\frac{6}{7}$	$59\frac{3}{7}$	64	$68\frac{4}{7}$	$73\frac{1}{7}$	$77\frac{5}{7}$	$82\frac{2}{7}$	$86\frac{6}{7}$	$91\frac{3}{7}$	96	$100\frac{4}{7}$
16	8	44	48	52	56	60	64	68	72	76	80	84	88
16	9	$39\frac{1}{9}$	$42\frac{2}{3}$	$46\frac{2}{9}$	$49\frac{7}{9}$	$53\frac{1}{3}$	$56\frac{8}{9}$	$60\frac{4}{9}$	64	$67\frac{5}{9}$	$71\frac{1}{9}$	$74\frac{2}{3}$	$78\frac{2}{9}$
16	10	$35\frac{1}{5}$	$38\frac{2}{5}$	$41\frac{3}{5}$	$44\frac{4}{5}$	48	$51\frac{1}{5}$	$54\frac{2}{5}$	$57\frac{3}{5}$	$60\frac{4}{5}$	64	$67\frac{1}{5}$	$70\frac{2}{5}$
17	7	$53\frac{3}{7}$	$58\frac{2}{7}$	$63\frac{1}{7}$	68	$72\frac{6}{7}$	$77\frac{5}{7}$	$82\frac{4}{7}$	$87\frac{3}{7}$	$92\frac{2}{7}$	$97\frac{1}{7}$	102	$106\frac{6}{7}$
17	8	$46\frac{3}{4}$	51	$55\frac{1}{4}$	$59\frac{1}{2}$	$63\frac{3}{4}$	68	$72\frac{1}{4}$	$76\frac{1}{2}$	$80\frac{3}{4}$	85	$89\frac{1}{4}$	$93\frac{1}{2}$
17	9	$41\frac{5}{9}$	$45\frac{1}{3}$	$49\frac{1}{9}$	$52\frac{8}{9}$	$56\frac{2}{3}$	$60\frac{4}{9}$	$64\frac{2}{9}$	68	$71\frac{7}{9}$	$75\frac{5}{9}$	$79\frac{1}{3}$	$83\frac{1}{9}$
17	10	$37\frac{2}{5}$	$40\frac{1}{5}$	$44\frac{1}{5}$	$47\frac{3}{5}$	51	$54\frac{2}{5}$	$57\frac{4}{5}$	$61\frac{1}{5}$	$64\frac{3}{5}$	68	$71\frac{2}{5}$	$74\frac{4}{5}$
18	7	$56\frac{4}{7}$	$61\frac{1}{7}$	$66\frac{6}{7}$	72	$77\frac{1}{7}$	$82\frac{2}{7}$	$87\frac{3}{7}$	$92\frac{4}{7}$	$97\frac{5}{7}$	$102\frac{6}{7}$	108	$113\frac{1}{7}$
18	8	$49\frac{1}{2}$	54	$58\frac{1}{2}$	63	$67\frac{1}{2}$	72	$76\frac{1}{2}$	81	85	90	$94\frac{1}{2}$	99
18	9	44	48	52	56	60	64	68	72	76	80	84	88
18	10	$39\frac{3}{5}$	$43\frac{1}{5}$	$46\frac{4}{5}$	$50\frac{2}{5}$	54	$57\frac{3}{5}$	$61\frac{1}{5}$	$64\frac{4}{5}$	$68\frac{2}{5}$	72	$75\frac{3}{5}$	$79\frac{1}{5}$
19	8	$52\frac{1}{4}$	57	$61\frac{3}{4}$	$66\frac{1}{2}$	$71\frac{1}{4}$	76	$80\frac{3}{4}$	$85\frac{1}{2}$	$90\frac{1}{4}$	95	$99\frac{3}{4}$	$104\frac{1}{2}$
19	9	$46\frac{4}{9}$	$50\frac{2}{3}$	$54\frac{8}{9}$	$59\frac{1}{9}$	$63\frac{1}{3}$	$67\frac{5}{9}$	$71\frac{7}{9}$	76	$80\frac{2}{9}$	$84\frac{4}{9}$	$88\frac{2}{3}$	$92\frac{8}{9}$
19	10	$41\frac{4}{5}$	$45\frac{3}{5}$	49	$53\frac{1}{5}$	57	$60\frac{4}{5}$	$64\frac{3}{5}$	$68\frac{2}{5}$	$72\frac{1}{5}$	76	$79\frac{4}{5}$	$83\frac{3}{5}$
20	8	55	60	65	70	75	80	85	90	95	100	105	110
20	9	$48\frac{8}{9}$	$53\frac{1}{3}$	$57\frac{7}{9}$	$62\frac{2}{9}$	$66\frac{2}{3}$	$71\frac{1}{9}$	$75\frac{5}{9}$	80	$84\frac{4}{9}$	$88\frac{8}{9}$	$93\frac{1}{3}$	$97\frac{7}{9}$
20	10	44	48	52	56	60	64	68	72	76	80	84	88

280. **Early Tricycle Chain.**—Figure 426 illustrates the 'Morgan' chain, used in some of the early tricycles, which was

composed of links made from round steel wire alternating with tubular steel rollers. There being only line contact between

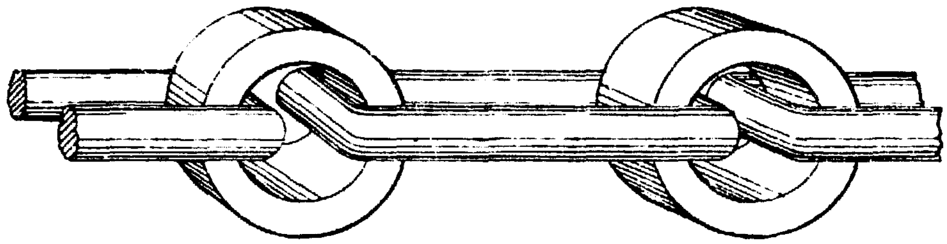


FIG. 426.

adjacent links and rollers, the wear was great, and this form of chain was soon abandoned.

281. **Humber Chain.**—Figure 427 shows the 'Humber' chain, formed by a number of hard steel blocks (fig. 428) alternating

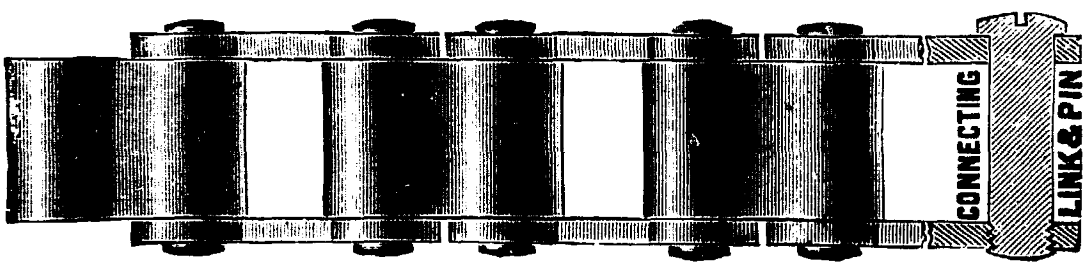


FIG. 427.

with side-plates (fig. 429). The side-plates are riveted together by a pin (fig. 430), which passes through the hole in the block. The rivet-pin is provided with shoulders at each end, so that the distance between the side-plates is preserved a trifle greater than

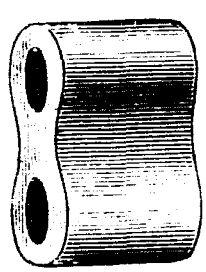


FIG. 428.

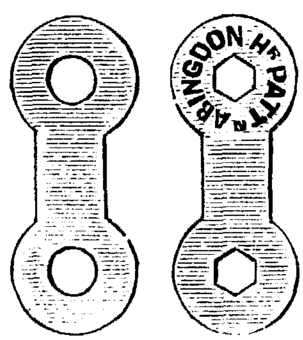


FIG. 429.

FIG. 430.

the width of the block. In the 'Abingdon-Humber' chain the holes in one of the side-plates are hexagonal, so that the pair of rivet-pins, together with the pair of side-plates they unite, form one rigid structure, and the pins are prevented from turning in the side-plates.

Figure 431 shows a 'Humber' pattern chain, made by Messrs. Perry & Co. The improvement in this consists principally in the addition of a pen steel bush surrounding the rivet. The ends of the bush are serrated, and its total length between the points is a trifle greater than the distance between the shoulders of the

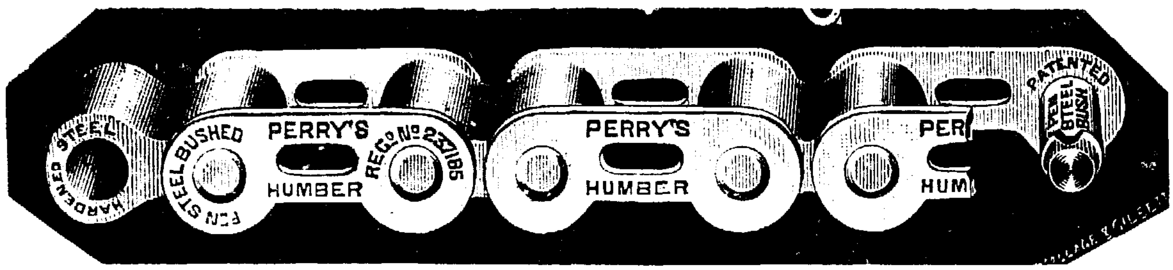


FIG. 431.

rivet-pin. The act of riveting thus rigidly fixes the bush to the side-plate, and prevents the rivet-pins turning in the side-plates. The hard pen-steel bush bears on the hard steel block, and there is, therefore, less wear than with a softer metal rubbing on the block.

Messrs. Brampton & Co. make a 'self-lubricating' chain of the 'Humber' type (fig. 432). The block is hollow, and made in

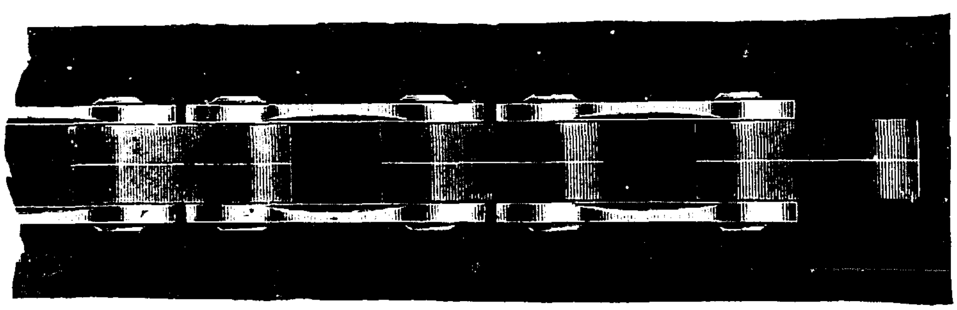


FIG. 432.

two pieces: the interior is filled with lubricant—a specially prepared form of graphite—sufficient for several years.

282. **Roller Chain.** Figure 433 shows a *roller* or *long-link chain*, as made by the Abingdon Works Co., the middle block of the 'Humber' chain being dispensed with, and the number of rivet-pins required being only one-half. Each chain-link is formed by two side-plates, symmetrically situated on each side of the centre line, and each rivet thus passes through four plates. The two outer plates are riveted together, forming one chain-link; while the two inner plates, forming the adjacent chain-link, can rotate

on the rivet-pin as a bearing. If the inner plates were left as narrow as the outer plates, the bearing surface on the rivet would

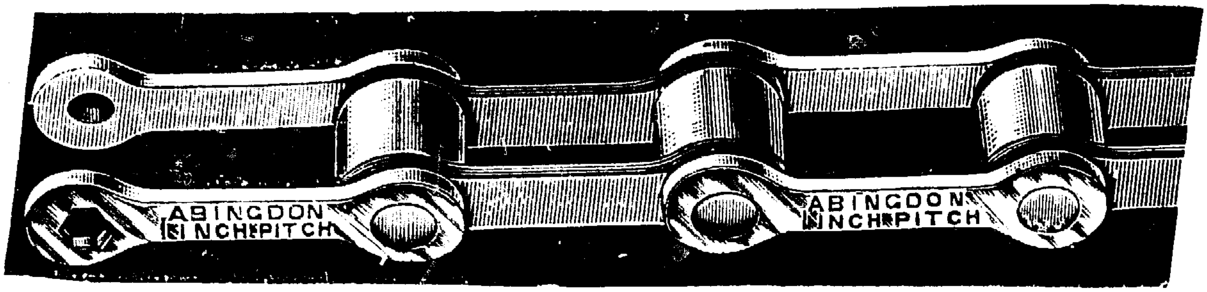


FIG. 433.

be very small, and wear would take place rapidly. Figure 434 shows the inner plate provided with bosses, so that the bearing surface is enlarged; and figure 435 shows the plates riveted together. The rivet, shown separately (fig. 436), thus bears along the whole width of the inner chain-link. Loose rollers surround the bosses; these are not shown in figure 435, but are shown in figure 433.

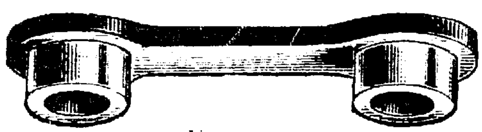


FIG. 434.

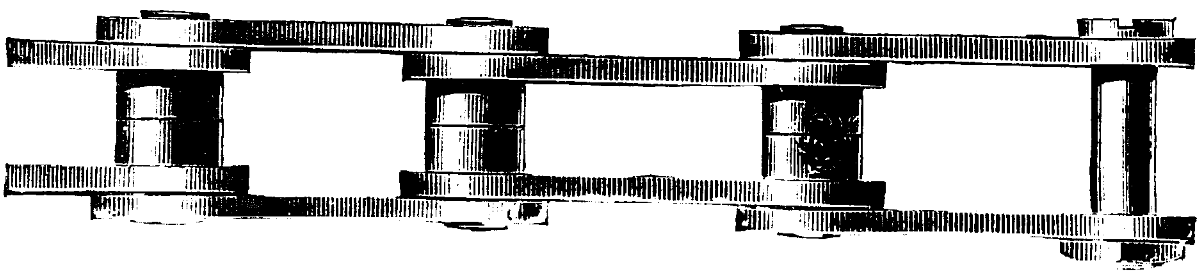


FIG. 435.

Single-link Chain.—The chain illustrated in figure 433 is a two-link chain; that is, its length must be increased or diminished by two links at a time. Thus, if the chain stretches and becomes too long for the cycle, it can only be shortened by two inches at a time. Figure 437 shows a *single-link chain*; that is, one which can be shortened by removing one link at a time. The side-plates in this case are not straight, but one pair of ends are brought closer together than the other; the details of boss, rivets, and rollers are the same as in the double-link chain.

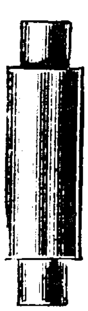


FIG. 436.

The width of the space between the side-plates of figure 433

on the rivet-pin as a bearing. If the inner plates were left as narrow as the outer plates, the bearing surface on the rivet would

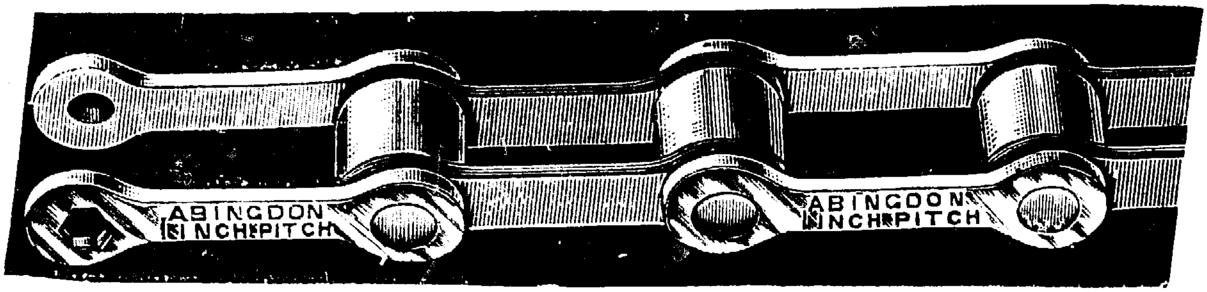


FIG. 433.

be very small, and wear would take place rapidly. Figure 434 shows the inner plate provided with bosses, so that the bearing surface is enlarged; and figure 435 shows the plates riveted together. The rivet, shown separately (fig. 436), thus bears along the whole width of the inner chain-link. Loose rollers surround the bosses; these are not shown in figure 435, but are shown in figure 433.

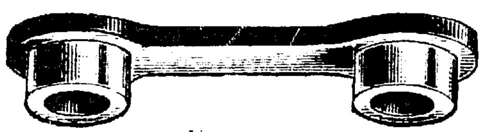


FIG. 434.

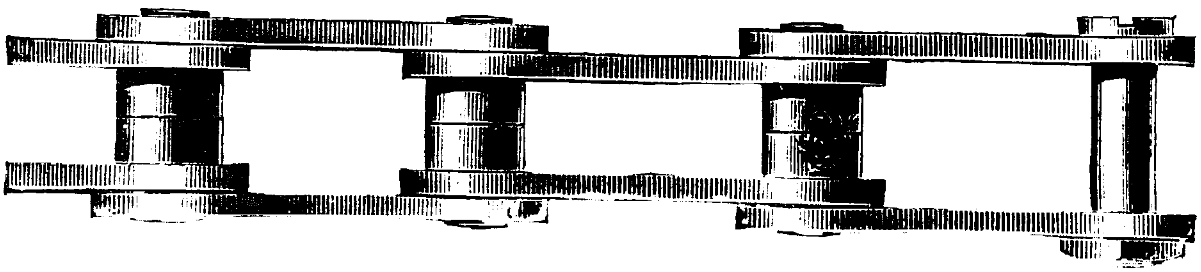


FIG. 435.

Single-link Chain.—The chain illustrated in figure 433 is a two-link chain; that is, its length must be increased or diminished by two links at a time. Thus, if the chain stretches and becomes too long for the cycle, it can only be shortened by two inches at a time. Figure 437 shows a *single-link chain*; that is, one which can be shortened by removing one link at a time. The side-plates in this case are not straight, but one pair of ends are brought closer together than the other; the details of boss, rivets, and rollers are the same as in the double-link chain.

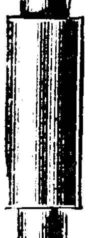


FIG. 436.

The width of the space between the side-plates of figure 433

there are teeth in the wheel. Let a, b, c . . . (fig. 440) be consecutive angles of the polygon. When the chain is wrapped round the wheel the centres of the chain rivets will occupy the positions a, b, c . . . The relative motion of the chain and wheel will be the same, if the wheel be considered fixed and the chain to be wound on and off. If the wheel be turning in the direction of the arrow, as the rivet a leaves contact with the wheel, it will move relative to the wheel in the circular arc $a\ a_1$, having b as centre, a_1 lying in the line $c\ b$ produced. Assuming that the chain is tight, the links $a\ b$ and $b\ c$ will now be in the same straight line, and the rivet a will move, relative to the chain-wheel, in the circular arc $a_1\ a_2$, with centre c; a_2 lying in the straight line $d\ c$

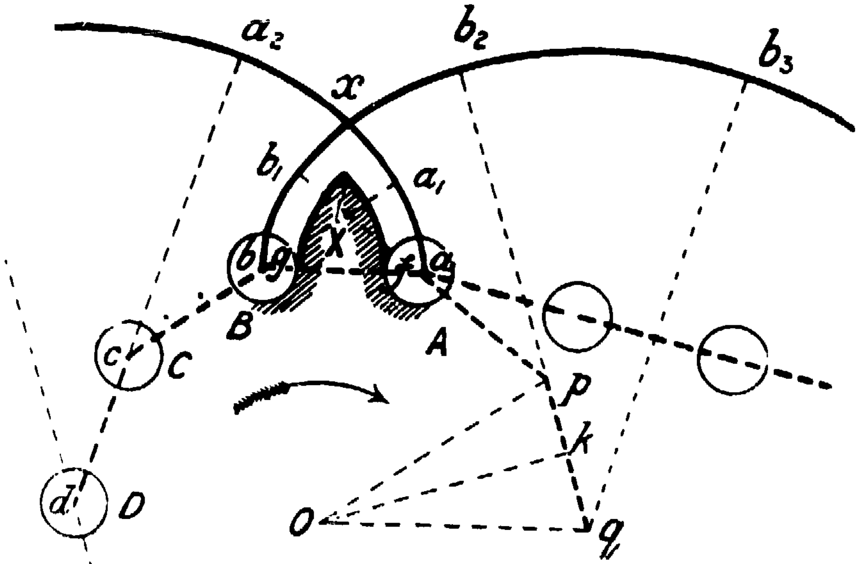


FIG. 440.

produced. Thus, the relative path of the centre of the rivet A as it leaves the wheel is a series of circular arcs, having centres b, c, d . . . respectively. It may be noticed that this path is approximately an involute of a circle, the approximation being closer the larger the number of teeth in the wheel. In the same way, the relative path of the centre of rivet b as it moves into contact with the wheel is an exactly similar curve $b\ b_1\ b_2$ . . ., which intersects the curve $a\ a_1\ a_2$ . . . at the point x. If the rivets and rollers of the chain could be made indefinitely small, the largest possible tooth would have the outline $a\ a_1\ x\ b_1\ b$. Taking account of the rollers actually used, the outline of the largest possible tooth will be a pair of parallel curves $A\ X$ and

BX intersecting at X, and lying inside $a\,a_1\ldots$ and $b\,b_1\ldots$, a distance equal to the radius of the rollers.

Kinematically there is no necessity for the teeth of a chain-wheel projecting beyond the pitch-line, as is absolutely essential in spur-wheel gearing. If the pitches of the chain and wheel could be made exactly equal, and the distance between the two chain-wheels so accurately adjusted that the slack of the chain could be reduced to zero, and the motion take place without side-swaying of the chain, the chain-wheel might be made as in figure 441. With this ideal wheel there would be no rubbing of the chain-links on it as they moved into and out of gear.

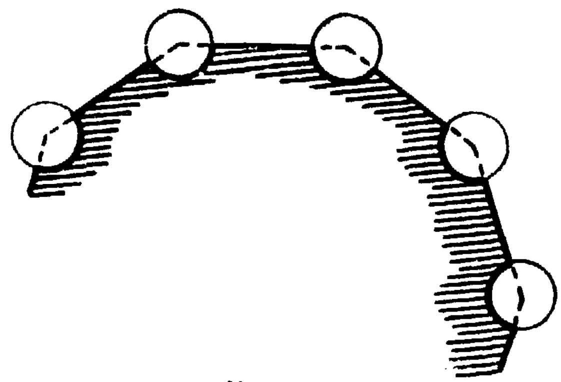


FIG. 441.

But, owing to gradual stretching, the pitch of the chain is seldom exactly identical with that of the wheel; this, combined with slackness and swaying of the chain, makes it desirable, and in fact necessary, to make the cogs project from the pitch-line. If the cogs be made to the outline AXB (fig. 440), each link of the chain will rub on the corresponding cog along its whole length as it moves into and out of gear; or rather, the roller may *roll* on the cog, and rub with its inner surface on the bosses of the inner plates of the link. To eliminate this rubbing the outline of the cog should therefore be drawn as follows: Let a and b (fig. 442) be two adjacent corners of the pitch-polygon, and let the rollers, with a and b as centres, cut $a\,b$ at f and g respectively. The centres m and n of the arcs of outline through f and g respectively should lie on $a\,b$, but closer together than a and b; in fact, f may conveniently be taken for the centre of the arc through g, and *vice versâ*. The addendum-circle may be conveniently drawn touching the straight line which touches, and lies entirely outside of, two adjacent rollers.

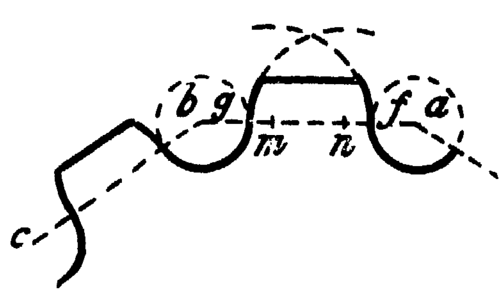


FIG. 442.

The 'Simpson' Lever-chain has triangular links, the inner corners, A, B, C . . . (fig. 443), are pin-jointed and gear in the ordinary way with the chain-wheel on the crank-axle. Rollers project from the outer corners, a, b, . . . and engage with the chain-wheel on the driving-hub. As the chain winds off the chain-wheel the relative path, $p\ p_1\ p_2$ . . . of one of the inner corners is, as in figure 440, a smooth curve made up of circular arcs, while that of an outer corner has cusps, a_1, a_2, . . . corre-

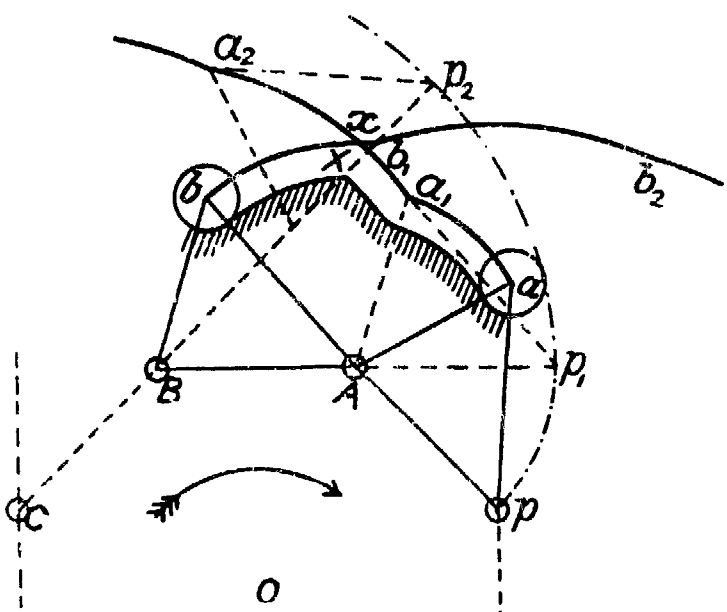


FIG. 443.

sponding to the sudden changes of the relative centre of rotation from A to B, from B to C, ... As the chain is wound on to the wheel, the relat path of an adjacent corner, b, is a curve, $b\ b_1\ b_2$ . . . of t.. same general character, but not of exactly the same shape, since the triangular links are not equal-sided. These two curves intersect at x, and the largest possible tooth outline is a curve parallel to $a\ a_1\ x\ b$. If the actual tooth outline lie a little inside this curve, as described in figure 442, the rubbing of the rollers on their pins will be reduced to a minimum, and the frictional resistance will not be greater than that of an ordinary roller chain. Thus there is no necessity for the cusp on the chain-wheel; the latter may therefore be made with a smaller addendum-circle.

Let a, p, and q (fig. 440) be three consecutive corners of the

pitch-polygon of a long-link chain-wheel, one-inch pitch. The circumscribing circle of the pitch-polygon may, for convenience of reference, be called the *pitch-circle.* Let R be the radius of the pitch-circle, and N the number of teeth on the wheel. From O, the centre, draw $O\,k$ perpendicular to $p\,q$. The angle $p\,O\,q$ is evidently $\frac{360}{N}$ degrees, and the angle $p\,O\,k$ therefore $\frac{180}{N}$ degrees. And

$$R = O\,p = \frac{p\,k}{\sin p\,O\,k} = \frac{0{\cdot}5}{\sin \frac{180^\circ}{N}} \text{ inches} \quad . \quad . \quad . \quad (3)$$

TABLE XV.—CHAIN-WHEELS, 1-IN. PITCH.

N Number of teeth in chain-wheel	R Radius of circumscribing circle of pitch-polygon: Long-link chain	R Radius of circumscribing circle of pitch-polygon: Humber chain	Radius of circle whose circumference is N inches
	Inches	Inches	Inches
6	1·000	·967	·955
7	1·153	1·125	1·114
8	1·307	1·283	1·274
9	1·462	1·441	1·433
10	1·618	1·599	1·592
11	1·775	1·758	1·751
12	1·932	1·916	1·910
13	2·089	2·074	2·069
14	2·247	2·233	2·228
15	2·405	2·392	2·387
16	2·563	2·551	2·546
17	2·721	2·710	2·705
18	2·880	2·870	2·865
19	3·039	3·029	3·024
20	3·197	3·188	3·183
21	3·356	3·347	3·342
22	3·514	3·505	3·501
23	3·672	3·664	3·660
24	3·831	3·824	3·820
25	3·990	3·983	3·979
26	4·148	4·142	4·138
27	4·307	4·301	4·297
28	4·466	4·460	4·456
29	4·626	4·620	4·616
30	4·785	4·779	4·775

The values of R for wheels of various numbers of teeth are given in Table XV.

285. **Humber Chain-wheel.**—The method of designing the form of the teeth of a 'Humber' chain-wheel is, in general, the same as for a long-link chain, the radius of the end of the hardened block being substituted for the radius of the roller; but the distance between the pair of holes in the block is different from that between the pair of holes in the side-plates, these distances

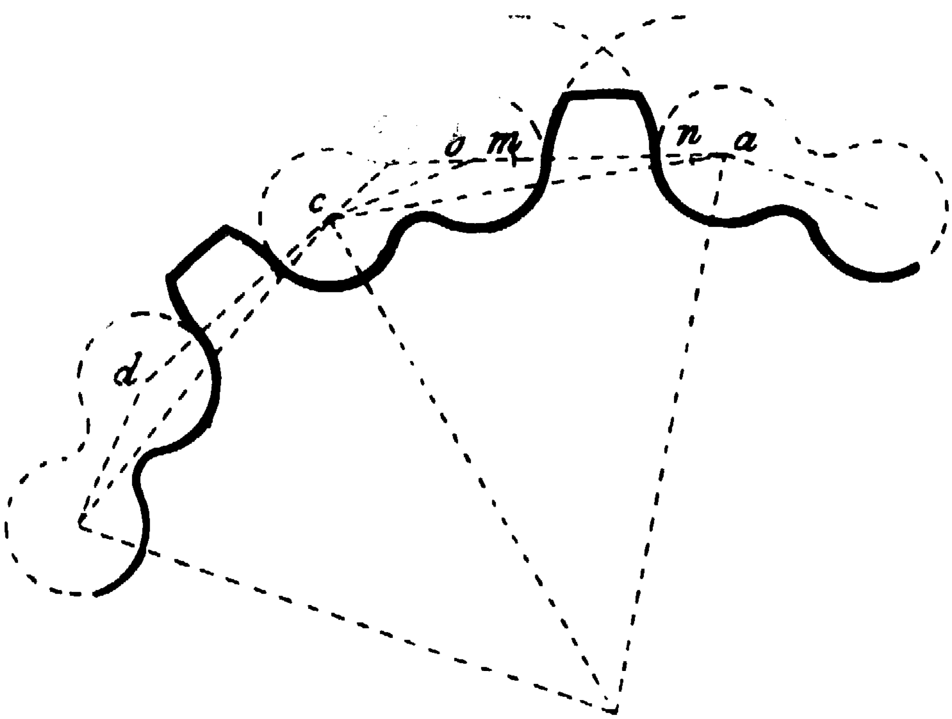


FIG. 444.

being approximately ·4 in. and ·6 in. respectively. The pitch-line of a 'Humber' chain-wheel will therefore be a polygon with its corners all lying on the circumscribing circle, but with its sides ·4 in. and ·6 in. long alternately. Figure 444 shows the method of drawing the tooth, the reference letters corresponding to those in figures 440 and 442, so that the instructions need not be repeated.

Let a, b, c, d (fig. 444) be four consecutive corners of the pitch-polygon of a 'Humber' chain-wheel. Produce the sides $a\,b$ and $d\,c$ to meet at e. Then, since a, b, c, and d lie on a circle, it is evident, from symmetry, that the angles $e\,b\,c$ and $e\,c\,b$ are equal. If N be the number of teeth in the wheel, there are $2\,N$

sides of the pitch-polygon, and the external angle $e\,b\,c$ will be $\frac{360}{2N} = \frac{180}{N}$ degrees.

Let $a\,c = D$, then $R = \dfrac{D}{2 \sin \frac{180^\circ}{N}}$.

But
$$D^2 = \overline{a\,b}^2 + \overline{b\,c}^2 + 2\,\overline{a\,b}\,.\,\overline{b\,c} \cos \frac{180^\circ}{N}$$
$$= \cdot 36 + \cdot 16 + \cdot 48 \cos \frac{180^\circ}{N}$$

$$\therefore\ R = \frac{\sqrt{\cdot 52 + \cdot 48 \cos \frac{180^\circ}{N}}}{2 \sin \frac{180^\circ}{N}} \qquad \qquad (4)$$

The radii of the pitch-circles of wheels having different numbers of teeth are given in Table XV.

286. **Side-clearance and Stretching of Chain.**—With chain-wheels designed as in sections 284–5, with the pitch of the teeth exactly the same as the pitch of the chain, there is no rubbing of the chain links on the wheel-teeth, the driving arc of action is the same as the arc of contact of the chain with the wheel, and all the links in contact with the wheel have a share in transmitting the effort. But when the pitch of the chain is slightly different from that of the wheel-teeth the action is quite different, and the chain-wheels should be designed so as to allow for a slight variation in the pitch of the chain by stretching, without injurious rubbing action taking place. The thickness of the teeth of the long-link chain-wheel (fig. 440) is so great that it can be considerably reduced without impairing the strength. Figure 445 shows a wheel in which the thickness of the teeth has been reduced. If the pitch of the chain be the same as that of the wheel, each tooth in the arc of contact will be in contact with a roller of the chain, and there will be a clearance space x between each roller and tooth. Let N be the number of teeth in the wheel; then the number of teeth in action will be in

general not more than $\frac{N}{2} + 1$. The original pitch of the chain may be made $\frac{x}{\frac{N}{2} + 1}$ *less* than the pitch of the wheel-teeth, the wheel and chain will gear perfectly together. Figure 445 illustrates the wheel and chain in this case. After a certain amount of wear and stretching, the pitch of the chain will become exactly the same as that of the teeth, and each tooth will have a roller in contact with it. The stretching may still continue until the pitch of the chain is $\frac{x}{\frac{N}{2} + 1}$ *greater* than that of the wheel-teeth, without any injurious action taking place.

The mutual action of the chain and wheel having different pitches must now be considered. First, let the pitch of the chain be a little *less* than that of the teeth (fig. 445), and suppose the

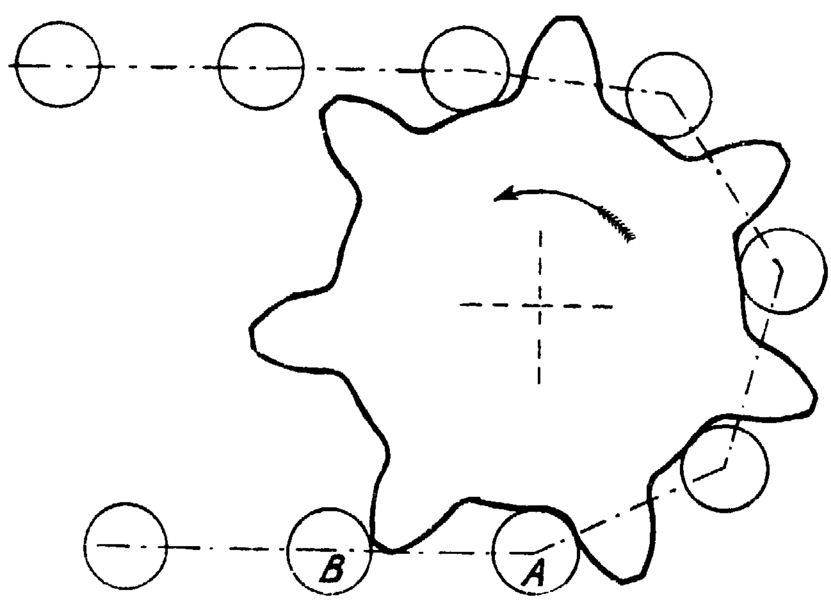


FIG. 445.

wheel *driven* in the direction of the arrow. One roller, *A*, just passing the lowest point of the wheel will be driving the tooth in front of it, and the following roller, *B*, will sooner or later come in contact with a tooth. Figure 445 shows the roller *B* just coming into contact with its tooth, though it has not yet reached the pitch-line of the wheel. The motion of the chain and wheel continuing, the roller *B* rolls or rubs on the tooth, and the

roller A gradually recedes from the tooth it had been driving. Thus the total effort is transmitted to the wheel by one tooth, or at most two, during the short period one roller is receding from, and another coming into, contact.

If the pitch of the chain be a little too great, and the wheel be *driven* in the same direction, the position of the acting teeth is at the top of the wheel (fig. 446). The roller, C, is shown driving the tooth in front of it, but as it moves outwards along the tooth surface the following roller, D, will gradually move up to, and drive, the tooth in front of it.

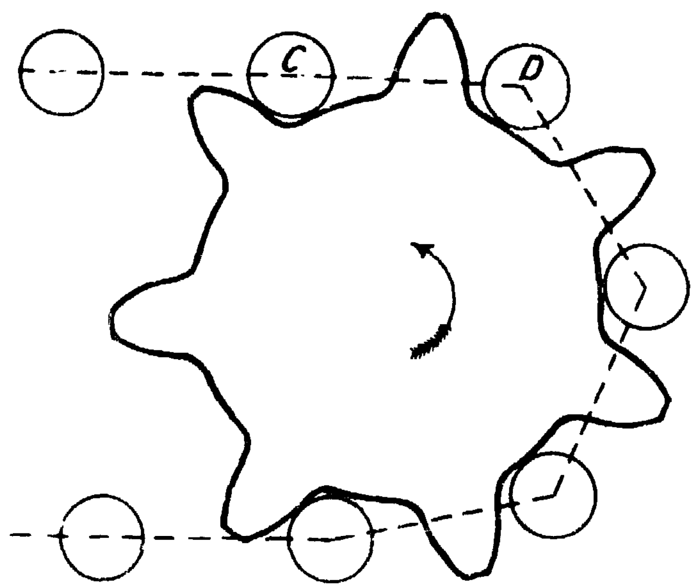


FIG. 446.

The action between the chain and the *driving*-wheel is also explained on the same general principles; if the direction of the arrow be reversed, figures 445 and 446 will illustrate the action.

In a chain-wheel made with side-clearance, assuming the pitches of chain and teeth equal, there will be two pitch-polygons for the two directions of driving. Let a and b (fig. 447) be two consecutive corners of one of the pitch-polygons, and let the roller

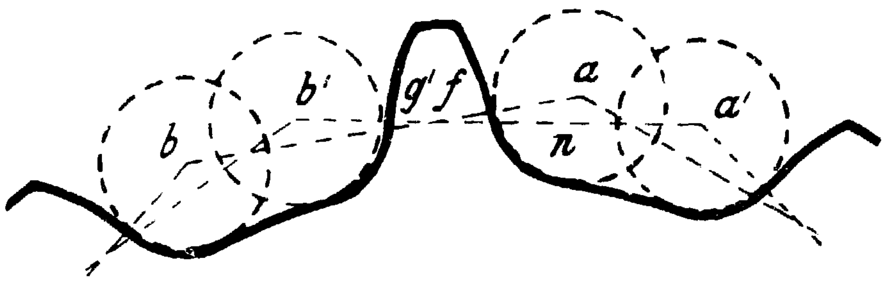


FIG. 447.

with centre a cut $a\,b$ at f. The centre m of the arc of tooth outline through f lies on $a\,b$. Let $a\,a' = b\,b'$ be the side-clearance measured along the circumscribing circle; a' and b' will therefore

with centre b' cut $a' b'$ in g; the centre n of the arc of tooth outline through f lies on $a' b'$. The bottom of the tooth space should be a circular arc, which may be called the root-circle, concentric with the pitch-polygons, and touching the circles of the rollers a and a'.

287. **Rubbing and Wear of Chain and Teeth.**—If the outline of the teeth be made exactly to the curve $f X$ (fig. 440), the roller A will knock on the top of the tooth, and will then roll or rub along its whole length. If the tooth be made to a curve lying inside $f X$, the roller will come in contact with the tooth at a point p (fig. 448), such that the distance of p from the curve $f X$ is equal to the difference of the pitches of the teeth and chain; $p f$ will be the arc of the tooth over which contact takes place. The length of this arc will evidently be smaller (and therefore also the less will be the work lost in friction), the smaller the radius of the tooth outline.

In the 'Humber' chain the block comes in contact with the teeth, and there is relative rubbing over the arc $p f$ (fig. 448).

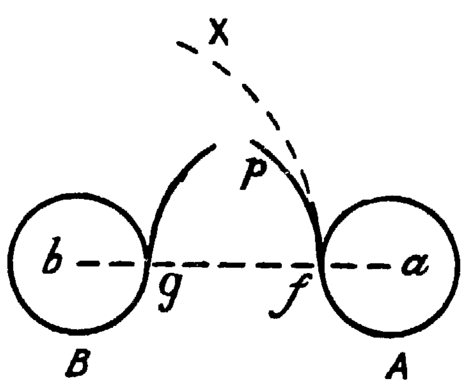


FIG. 448.

The same point of the block always comes in contact with the teeth, so that after a time the wear of the blocks of the chain and the teeth of the wheel becomes serious, especially if the wheel-teeth be made rather full.

The chief advantage of a roller-chain lies in the fact that the roller being free to turn on the rivet, different points of the roller come successively in contact with the wheel-teeth. If the chain be perfectly lubricated the roller will actually roll over its arc of contact, $f p$, with the tooth, and will rub on its rivet-pin. The rubbing is thus transferred from a higher pair to a lower pair, and the friction and wear of the parts, other things being equal, will be much less than in the 'Humber' chain. Even with imperfect lubrication, so that the roller may be rather stiff on its rivet-pin, and with rubbing taking place over the arc $f p$, the roller will at least be slightly disturbed in its position relative to its rivet-pin, and a fresh portion of it will next come in contact with the wheel-teeth. Thus, even under the most unfavourable conditions, the

wear of the chain is distributed over the cylindrical surface of the roller, consequently the alteration of form will be much less than in a 'Humber' chain under the same conditions.

It must be clearly understood that the function performed by rollers in a chain is quite different from that in a roller-bearing. In the latter case rubbing friction is eliminated, but not in the former.

288. **Common Faults in Design of Chain-wheels.**—The portions of the teeth lying outside the pitch-polygon are often made far too full, so that a part of the tooth lies beyond the circular arc fX (fig. 440); the roller strikes the corner of the tooth as it comes into gear, and the rubbing on the tooth is excessive. This faulty tooth is illustrated in figure 449.

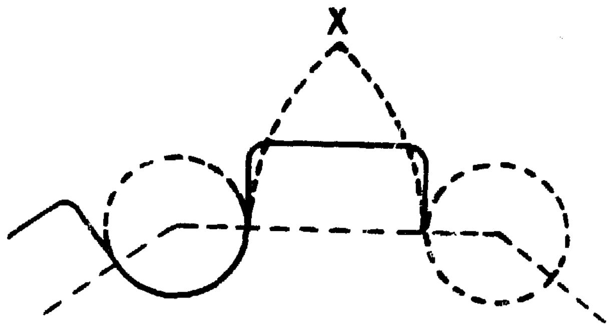


FIG. 449.

In long-link chain-wheels the only convex portion is very often merely a small circular arc rounding off the side of the tooth into the addendum-circle of the wheel. This rounding off of the corner is very frequently associated with the faulty design above mentioned. If the tooth outline be made to fp, a curve lying well within the circular arc fX (fig. 448), this rounding off of the corners of the teeth is quite unnecessary.

Another common fault in long-link chain-wheels is that the bottom of the tooth space is made one circular arc of a little larger radius than the roller. There is in this case no clearly defined circle in which the centres of the rollers are compelled to lie, unless the ends of the link lie on the cylindrical rim from which the teeth project. In back-hub chain-wheels this cylindrical rim is often omitted. Care should then be taken that the tooth space has a small portion made to a circle concentric with the pitch-circle of the wheel. Again, in this case, the direction of the mutual force between

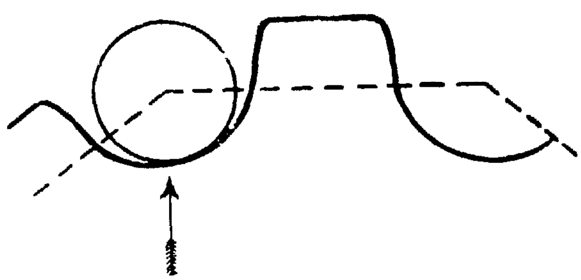


FIG. 450.

the roller and wheel is not along the circumference of the pitch-polygon ; there is therefore a radial component tending to force the rollers out of the tooth spaces, that is, there is a tendency of the chain to mount the wheel (fig. 450).

In 'Humber' pattern chain-wheels the teeth are often quite straight (fig. 451). This tooth-form is radically wrong. If the teeth are so narrow at the top as to lie inside the curve fX, the force acting on the block of the chain will have an outward component, and the chain will tend to mount the wheel. This faulty design is sometimes carried to an extreme by having the teeth *concave* right to the addendum-circle.

FIG. 451.

Either of the two faults above discussed gives the chain a tendency to mount the wheel, and this tendency will be greater the more perfect the lubrication of the chain and wheel.

289. **Summary of conditions determining the proper form of Chain-wheels.**—1. Provision should be made for the gradual stretching of the chain. This necessitates the gap between two adjacent teeth being larger than the roller or block of the chain.

2. The centres of the rollers in a long-link chain, or the blocks in a 'Humber' chain, must lie on a perfectly defined circle concentric with the chain-wheel. When the wheel has no distinct cylindrical rim, the bottom of the tooth space must therefore be a circular arc concentric with the pitch-polygon.

3. In order that there should be no tendency of the chain to be forced away from the wheel, the point of contact of a tooth and the roller or block of the chain should lie on the side of the pitch-polygon, and the surface of the tooth at this point should be at right angles to the side of the pitch-polygon. The centre of the circular arc of the tooth outline must therefore lie on the side of the pitch-polygon.

4. The blocks or rollers when coming into gear must not strike the corners of the teeth. The rubbing of the roller or block on the tooth should be reduced to a minimum. Both these conditions determine that the radius of the tooth outline should be less than 'length of side-plate of chain, minus radius of roller or block.'

The following method of drawing the teeth is a *résumé* of the results of sections 285–8, and gives a tooth form which satisfies the above conditions: Having given the type of chain, pitch, and number of teeth in wheel, find R, the radius of the pitch-circle $c\,c$, by calculation or from Table XV. On the pitch-circle $c\,c$ (fig. 452), mark off adjacent corners a and b of the pitch-polygon. With centres a and b, and radius equal to the radius of the roller (or the radius of the end of the block in a 'Humber' chain), draw circles, that from a as centre cutting $a\,b$ at f. Through f draw a circular arc, $f\,h$, with centre m on $a\,b$, $m\,f$ being less than $b\,f$. Mark off,

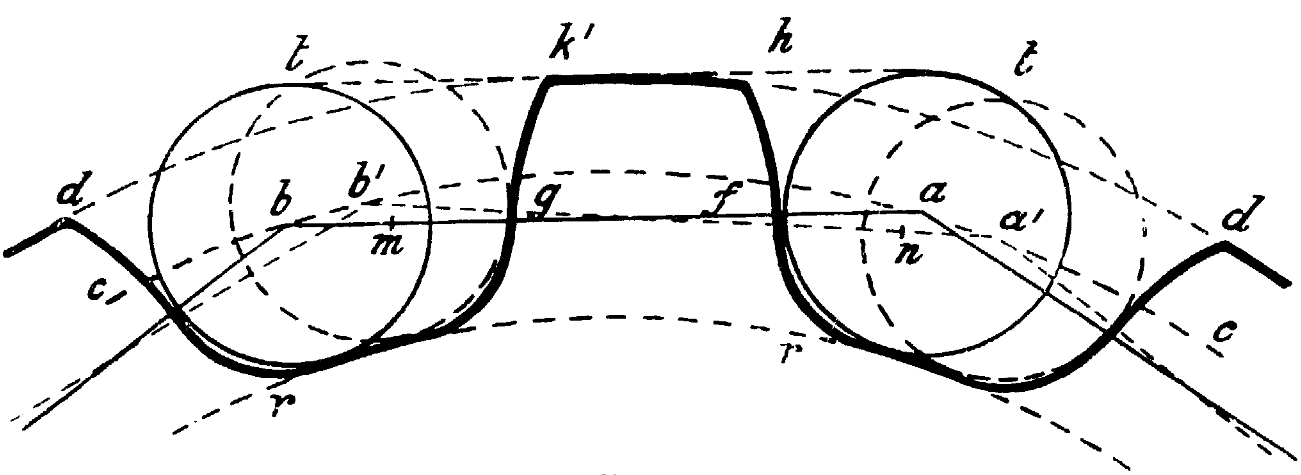


FIG. 452.

along the circle $c\,c$, $a\,a^1 = b\,b^1$ = side clearance required, and with centres a^1 and b^1, and the same radius as the rollers, draw circles, that from centre b^1 cutting the straight line $a^1\,b^1$ at g. With centre n^1 lying on $a^1\,b^1$, and radius equal to $m\,f$, draw a circular arc $g\,k^1$. Draw the root-circle $r\,r$ touching, and lying inside, the roller circles. The sides $f\,h$ and $g\,k^1$ of the tooth should be joined to the root-circle $r\,r$ by fillets of slightly smaller radius than the rollers. Draw a common tangent $t\,t$ to the roller circles a and b, and lying outside them; the addendum-circle may be drawn touching $t\,t$.

It should be noticed that this tooth form is the same whatever be the number of teeth in the wheel, provided the side-clearance be the same for all. The form of the spaces will, however, vary with the number of teeth in the wheel. A single milling-cutter to cut the two sides of the same tooth might herefore serve for all sizes of wheels; whereas when the milling-cutter cuts out the

space between two adjacent teeth, a separate cutter is required for each size of wheel.

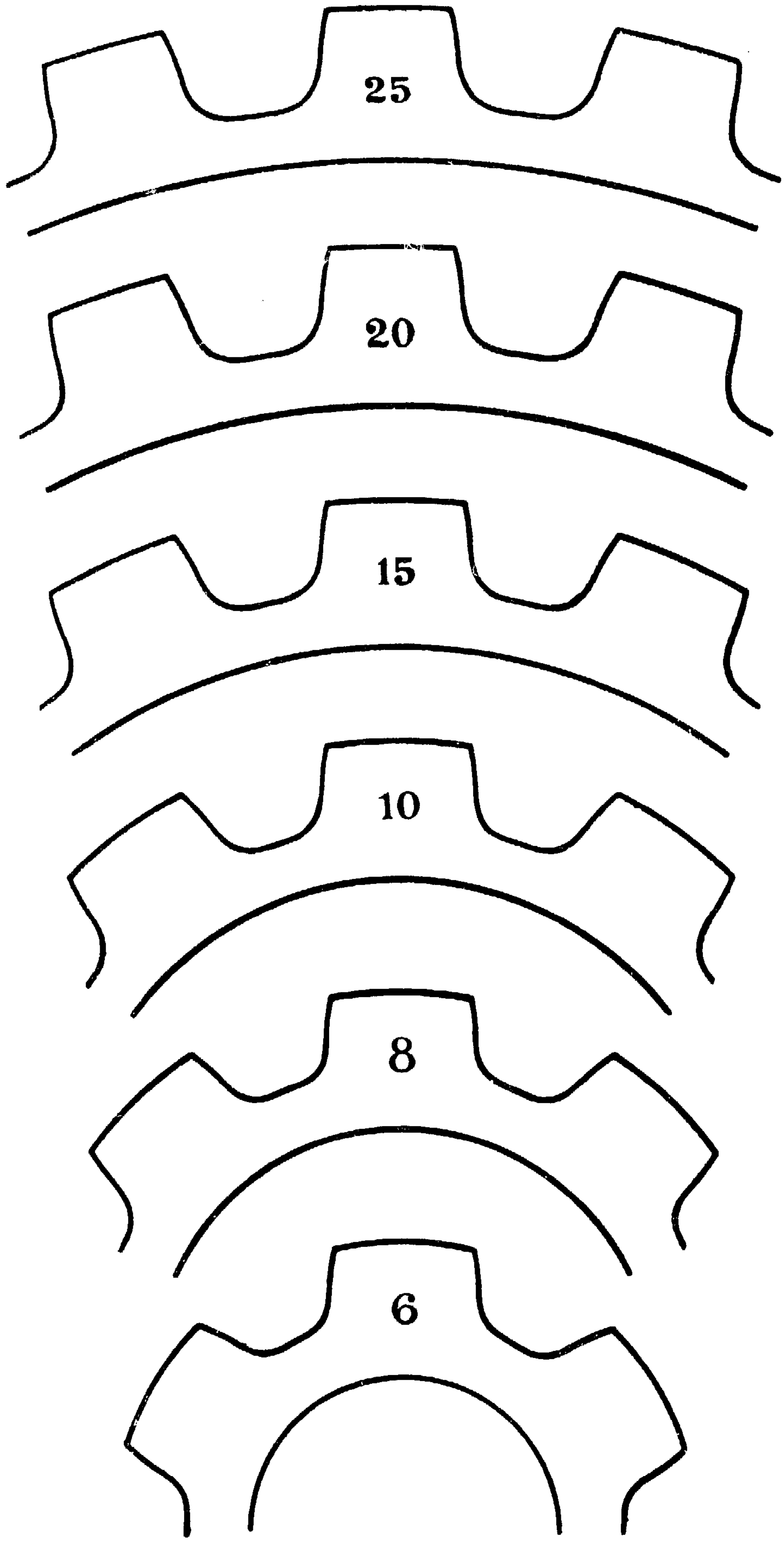


FIG. 453.

Figure 453 shows the outlines of wheels for inch-pitch long-links made consistent with these conditions, the diameter of the roller being taken $\frac{3}{8}$ in. The radius of the side of the tooth is in

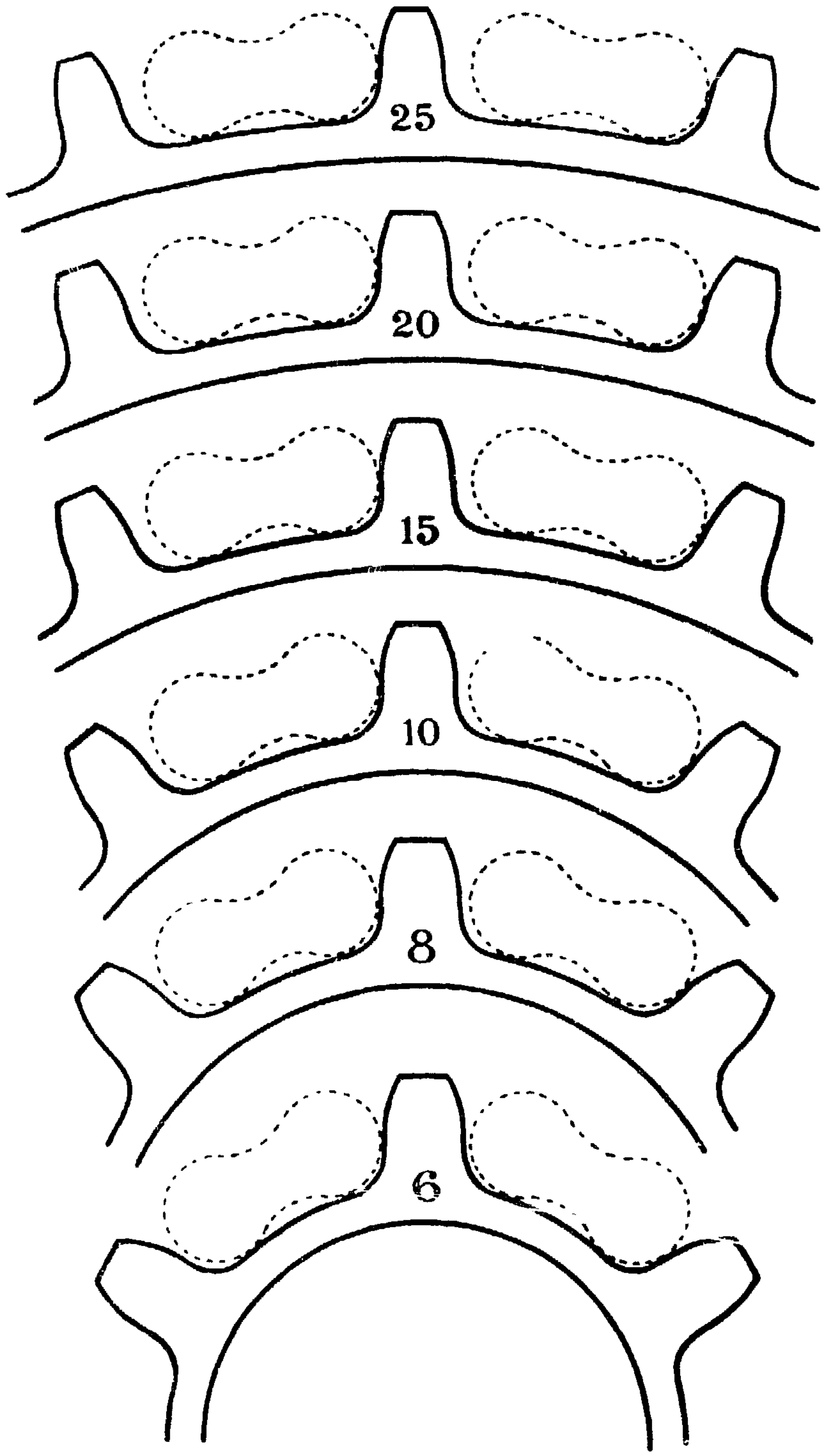


FIG. 454.

each case $\frac{5}{8}$ in. (it may with advantage be taken less), and the radius of the fillet at root of tooth $\frac{1}{8}$ in. The width of the roller space measured on the pitch-polygon is ($\cdot 375 + \cdot 005\ N$) in. ; N being the number of teeth in the wheel.

Figure 454 shows the outlines of wheels for use with the 'Humber' chain, the pitch of the rivet-pins in the side-plates being ·6 in. and in the blocks ·4 in., and the ends of the blocks being circular, ·35 in. diameter.

290. **Section of Wheel Blanks.**—If the chain sways sideways, the side-plates may strike the tops of the teeth as they come into gear, and cause the chain to mount the wheel, unless each link is properly guided sideways on to the wheel. The cross section of the teeth is sometimes made as in figure 454A, the sides being parallel and the top corners rounded off. A much better form of section, which will allow of a considerable amount of swaying without danger, is that shown in figure 454B. The thickness of the tooth at the root is a trifle less than the width of the space between the side-plates of the link. The thickness at the point is very small—say, $\frac{1}{32}$ in. to $\frac{1}{16}$ in.—and the tooth section is a wedge with curved sides.

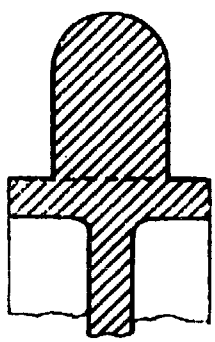


FIG. 454A.

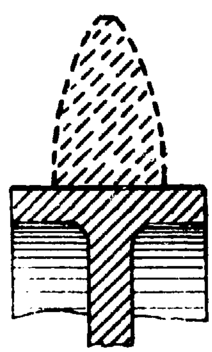


FIG. 454B.

If the side-plates of the chain be bevelled, as in Brampton's bevelled chain (fig. 432), an additional security against the chain coming off the wheel through side swaying will be obtained.

291. **Design of Side-plates of Chain.**—The side-plates of a well-designed chain should be subjected to simple tension. If P be the total pull on the chain, and A the least sectional area of the two side-plates, the tensile stress is $\frac{P}{A}$. Such is the case with side-plates of the form shown in figure 429.

Example I.—The section of the side-plates (fig. 429) is ·2 in deep and ·09 in. thick. The total sectional area is thus

$$2 \times \cdot 2 \times \cdot 09 = \cdot 036 \text{ sq. in.}$$

The proof load is 9 cwt. = 1,008 lbs. The tensile stress is, therefore,

$$f = \frac{1{,}008}{\cdot 036} = 28{,}000 \text{ lbs. per sq. in.}$$

$$= 12 \cdot 5 \text{ tons per sq. in.}$$

A considerable number of chains are being made with the side-plates recessed on one side, and not on the other (fig. 455). These side-plates are subjected to combined tensile and bending stresses. Let b be the width of the plate, t its thickness, b_1 be the depth of the recess, and let $b_2 = b - b_1$; that is, b_2 would be the width of the plate if recessed the same amount on both sides. The distance of the centre of the section from the centre line joining the rivets is $\frac{b_1}{2}$.

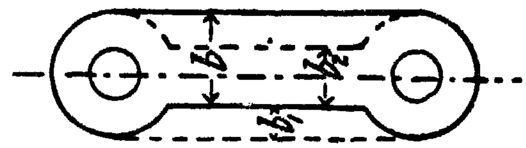


FIG. 455.

The bending-moment M on the link is $\frac{P b_1}{2}$. The modulus of the section, Z, is $\frac{2\,t\,b^2}{6}$. The maximum tensile stress on the section is (sec. 101)

$$f = \frac{P}{A} + \frac{M}{Z} = \frac{P}{2\,b\,t} + \frac{6\,P b_1}{2.\,2\,t b^2}$$

$$= \frac{P}{2\,b\,t}\left\{1 + \frac{3\,b_1}{b}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The stress on the side-plate if recessed on both sides would be

$$f = \frac{P}{2\,(b - b_1)\,t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The stress f calculated from equation (6) is always, within practical limits, less than the stress calculated from equation (5); and, therefore, the recessed side-plates can be actually strengthened by cutting away material. This can easily be proved by an elementary application of the differential calculus to equation (5).

Example II.—Taking a side-plate in which $t = \cdot 09$ in., $b = \cdot 3$ in., $b_1 = \cdot 1$ in., and therefore $b_2 = \cdot 2$ in., we get

$$A = 2 \times \cdot 09 \times \cdot 3 = \cdot 054 \text{ sq. in.},$$

and

$$Z = \frac{2 \times \cdot 09 \times \cdot 3^2}{6}$$

$$= \cdot 0027 \text{ in.}^3$$

The distance of the centre of the section from the centre-line of

the side-plate is ·05 in., and the bending-moment under a proof load of 9 cwt. is

$$M = 9 \times 112 \times \cdot 05 = 50 \cdot 4 \text{ inch-lbs.}$$

The maximum stress on the section is

$$f = \frac{1008}{\cdot 054} + \frac{50 \cdot 4}{\cdot 0027} = 37{,}320 \text{ lbs. per sq. in.}$$
$$= 16 \cdot 7 \text{ tons per sq. in.}$$

Thus this link, though having 50 per cent. more sectional area, is much weaker than a link of the form shown in figure 429.

If the plate be recessed on both sides, $A = \cdot 036$ sq. in., and

$$f = \frac{1008}{\cdot 036} = 28{,}000 \text{ lbs. per sq. in.}$$
$$= 12 \cdot 5 \text{ tons per sq. in.}$$

Thus the side-plate is strengthened, even though 33 per cent. of its section has been removed.

From the above high stresses that come on the side-plates of a chain during its test, and from the fact that these stresses may occasionally be reached or even exceeded in actual work when grit gets between the chain and wheel, it might seem advisable to make the side-plates of steel bar, which has had its elastic limit artificially raised considerably above the stresses that will come on the links under the proof load.

The Inner Side-plates of a Roller Chain, made as in figure 434, are also subjected to combined tension and bending in ordinary working. Assuming that the pressure between the rivet and inner link is uniformly distributed, the side-plate of the latter will be subjected to a bending-moment $M = \frac{P}{2} \times l$, l being the distance measured parallel to the axis of the rivet, between the centres of the side-plate and its boss, respectively.

Example III.—Taking $l = \cdot 08$ in., and the rest of the data as in the previous example, the maximum additional stress on the side-plate due to bending is

$$\frac{M}{Z} = \frac{504 \times \cdot 08 \times 6}{\cdot 2 \times \cdot 0081} = 149{,}000 \text{ lbs. per sq. in.}$$
$$= 66 \cdot 7 \text{ tons per sq. in.,}$$

which, added to the 12·5 tons per sq. in. due to the direct pull, gives a total stress of 79·2 tons per sq. in. Needless to say, the material cannot endure such a stress; what actually happens during the test is, the side-plates slightly bend when the elastic limit is reached, the pressure on the inner edge of the boss is reduced, so that the resultant pressure between the rivet and side-plate acts nearly in line with the latter. Thus the extra bearing surface for the rivet, supposed to be provided by the bosses, is practically got rid of the first time a heavy pull comes on the chain.

A much better method of providing sufficient bearing surface for the rivet-pins is to use a tubular rivet to unite the inner side-plates (fig. 456), inside which the rivet-pin uniting the outer side-plates bears, and on the outside of which the roller turns. This is the method adopted by Mr. Hans Renold for large gearing chains.

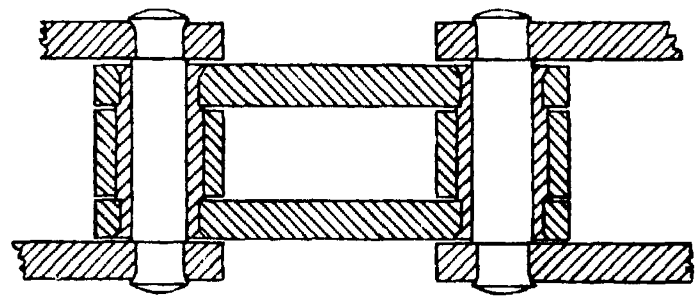


FIG. 456.

Side-plates of Single-link Chain.—In the same way, it will be readily seen that the maximum stress on the side-plates of the chain shown in figure 437 is much greater than on a straight plate with the same load. If the direction of the pull on the plate be parallel to the centre line of the chain, each plate will be subjected to a bending action. The transverse distance between the centres of the sections of the two ends is t, the bending-moment on the section will therefore be Pt. A more favourable assumption will be that the pull on each plate will be in a line joining the middle points of its ends. The greatest distance between this line and the middle of section will be then nearly $\frac{t}{2}$, and $M = \frac{Pt}{2}$. The bending in this case is in a plane at right angles to the direction of the bending in the recessed side-plate (fig. 455). The modulus of the section Z is $\frac{bt^2}{6}$.

Example IV.—Taking the same data as in the former examples,

the load on the chain is 9 cwt., $b = \cdot 3$ in., and $t = \cdot 09$ in., the pull on each plate is 504 lbs., the bending-moment is $\frac{504 \times \cdot 09}{2}$ $= 22 \cdot 7$ inch-lbs., $A = \cdot 027$ sq. in., $Z = \frac{\cdot 3 \times \cdot 09^2}{6} = \cdot 000405$ in.^{3} The maximum stress on the section is therefore

$$f = \frac{P}{A} + \frac{M}{Z}$$

$$a = \frac{504}{\cdot 027} + \frac{22 \cdot 7}{\cdot 000405}$$

$$= 18,670 + 56,050 = 74,720 \text{ lbs. per sq. in.}$$

$$= 33 \cdot 3 \text{ tons per sq. in.}$$

292. **Rivets.**—The pins fastening together the side-plates must be of ductile material, so that their ends may be riveted over without injury. A soft ductile steel has comparatively low tensile and shearing resistances. The ends of these pins are subjected to shearing stress due to half the load on the chain. If d be the diameter of the rivet, its area is $\frac{\pi d^2}{4}$, and the shearing stress on it will be

$$\frac{2P}{\pi d^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Example I.—If the diameter of the holes in the side-plate (fig. 429) be $\cdot 15$ in., under a proof load of 9 cwt. the shearing stress will be

$$\frac{1008}{2 \times \cdot 01767} = 28,500 \text{ lbs. per sq. in.}$$

$$= 12 \cdot 78 \text{ tons per sq. in.}$$

The end of the rivet is also subjected to a bending-moment $\frac{P}{2} \times \frac{t}{2}$. The modulus of the circular section is approximately $\frac{d^3}{10}$, the stress due to bending will therefore be

$$f = \frac{10 Pt}{4 d^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Example II.—Taking the dimensions in Example I. of section 291, and substituting in (8), the stress due to bending is

$$f = \frac{10 \times 1008 \times \cdot 09}{4 \times \cdot 15^3} = 67{,}200 \text{ lbs. per sq. in.}$$
$$= 30 \text{ tons per sq. in.}$$

The rivet is thus subjected to very severe stresses, which cause its ends to bend over (fig. 457).

The stretching of a chain is probably always due more to the yielding of the rivets than to actual stretching of the side-plates, if the latter are properly designed. A material that is soft enough to be riveted cold has not a very high tensile or shearing resistance. It would seem advisable, therefore, to make the pins of hard steel with a very high elastic limit, their ends being turned down with slight recesses (fig. 458), into which the side plates, made of a softer steel, could be forced by pressure. The Cleveland Cycle Company, and the Warwick and Stockton Company, manufacture chains on this system.

FIG. 457.

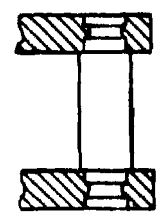


FIG. 458.

293. **Width of Chain and Bearing Pressure on Rivets.**—In the above investigations it will be noticed that the width of the chain does not enter into consideration at all. The only effect the width of the chain has is on the amount of bearing surface of the pins on the block. If l be the width of the block, and d the diameter of the pin, the projected bearing area is ld, and the intensity of pressure is $\frac{P}{dl}$. If the diameter of the pin (fig. 430) be $\cdot 17$ in., and the width of the block be $\frac{5}{16}$ in. $= \cdot 3125$ in., the bearing pressure under the proof load will be

$$\frac{1008}{\cdot 17 \times \cdot 3125} = 18{,}980 \text{ lbs. per sq. in.}$$

This pressure is very much greater than occurs in any other example of engineering design. Professor Unwin, in a table of 'Pressures on Bearings and Slides,' gives 3,000 lbs. per sq. in. as the maximum value for bearings on which the load is inter-

mittent and the speed slow. Of course, in a cycle chain the period of relative motion of the pin on its bearing is small compared to that during which it is at rest, so that the lubricant, if an oil-tight gear-case be used, gets time to find its way in between the surfaces.

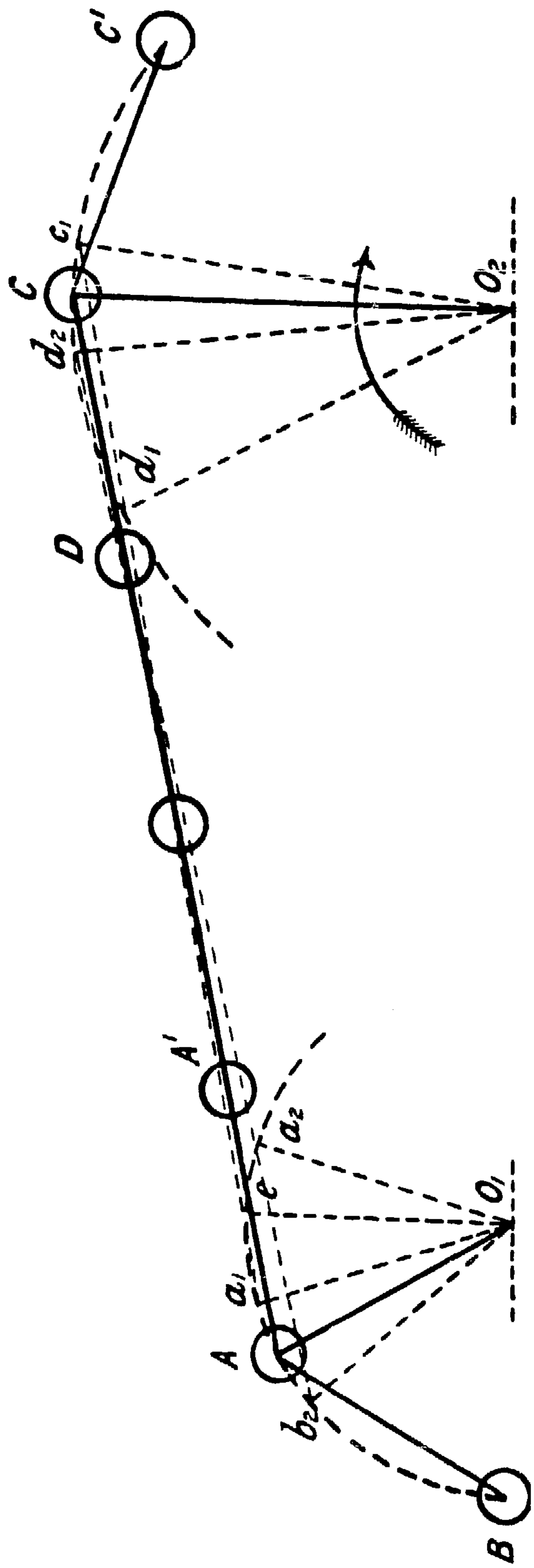


FIG. 459.

294. **Speed-ratio of Two Shafts Connected by Chain Gearing.**—The *average* speeds of two shafts connected by chain gearing are inversely proportional to the numbers of teeth in the chain wheels; but the speed-ratio is not *constant*, as in the case of two shafts connected together by a belt or by toothed-wheels. Let O_1 and O_2 (fig. 459) be the centres of the two shafts, let the wheel O_2 be the driver, the motion being as indicated by the arrow, and let $A C$ be the straight portion of the chain between the wheels at any instant. The instantaneous angular speed-ratio of the wheels is the same as that of two cranks $O_1 A$ and $O_2 C$ connected by the coupling-rod $A C$. Let B and D be the rivets consecutive to A and C respectively; then, as the motion of the wheels continues, the rivet D will ultimately touch the chain-wheel at the point $d_1 - a_1$, d'_1 and c_1 being in the same straight line—and the angular speed-ratio of the wheels will be the same as that of the two cranks $O_1 A$ and $O_2 D$ connected by the straight coup-

ling-rod $A D$, shorter by one link than the coupling-rod $A C$. The motion continuing, the rivet A leaves contact with the chain-wheel at a_2, and the virtual coupling-rod becomes $B D$; the points b_2, a_2, and d_2 lying in one straight line. The angular speed-ratio of the wheels is now the same as that of the two cranks $O_1 B$ and $O_2 D$ connected by the coupling-rod $B D$, of the same length as $A C$.

Thus, with a long-link chain, the wheels are connected by a virtual coupling-rod whose length changes twice while the chain moves through a distance equal to the length of one of its links. The small chain-wheel, being rigidly connected to the driving-wheel of the bicycle, will rotate with practically uniform speed; since the whole mass of the machine and rider acts as an accumulator of energy (or fly-wheel), keeping the motion steady. The chain-wheel on the crank-axle will therefore rotate with variable speed. The speed-ratio in any position, say $O_1 A C O_2$, can be found, after the method of section 32, by drawing $O_1 e$ parallel to $O_2 C$, meeting $C A$ (produced if necessary) at e; the intercept $O_1 e$ is proportional to the angular speed of the crank-axle. If this length be set off along $O_1 A$, and the construction be repeated, a polar curve of angular speed of the crank-axle will be obtained. It will be noticed that in figure 459 the angular speed of the crank-axle decreases gradually shortly after passing the position $O_2 d_1$ until the position $O_2 d_2$ is reached, and the rivet A attains the position a_2. The length of the coupling-rod being now increased by one link, the angular speed of the crank-axle increases gradually until the rivet C attains the position c_1. Here the length of the coupling-rod is decreased by one link, and the virtual crank of the wheel changing suddenly from $O_1 c_1$ to $O_2 d_1$, the length of the intercept $O e$ also changes suddenly, corresponding to a sudden change in the angular speed of the crank-axle.

With a 'Humber' chain the speed will have four maximum and minimum values while the chain moves over a distance equal to one link.

The magnitude of the variation of the angular speed of the crank-axle depends principally, as an inspection of figure 459 will show, on the number of teeth in the smaller wheel. If the crank-

axle be a considerable distance from the centre of the driving-wheel, and if the number of teeth of the wheel on the crank-axle be great, the longest intercept, $O\,e$, will be approximately equal to the radius of the pitch-circle, and the smallest intercept to the radius of the inscribed circle of the pitch-polygon. The variation of the angular speed of the crank can then be calculated as follows:

Let N_1 and N_2 be the numbers of teeth in the chain-wheels on the driving-hub and crank-axle respectively, R_1 and R_2 the radii of the pitch-circles, r_1 and r_2 the radii of the *inscribed* circles of the pitch-polygon. Then for a long-link chain the average speed-ratio $= \frac{N_1}{N_2}$. Assuming the maximum intercept $O_1 e$ (fig. 459) to be equal to R_1, then from (3), the

$$\text{maximum speed-ratio} = \frac{R_1}{R_2} = \frac{\sin\frac{\pi}{N_2}}{\sin\frac{\pi}{N_1}} \text{ approx.} \quad . \quad (9)$$

Assuming the minimum intercept $O\,e$ (fig. 459) to be equal to r_1, then from (3)

$$\text{minimum speed-ratio} = \frac{r_1}{R_2} = \frac{\sin\frac{\pi}{N_2}}{\tan\frac{\pi}{N_1}} \text{ approx.} \quad . \quad (10)$$

Then, for the crank-axle,

$$\frac{\text{maximum speed}}{\text{mean speed}} = \frac{N_2 \sin\frac{\pi}{N_2}}{N_1 \sin\frac{\pi}{N_1}} \text{ approx.} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$\frac{\text{minimum speed}}{\text{mean speed}} = \frac{N_2 \sin\frac{\pi}{N_2}}{N_1 \tan\frac{\pi}{N_1}} \text{ approx.} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$\frac{\text{maximum speed}}{\text{minimum speed}} = \frac{Op}{Ok} \text{ (fig. 440)} = \frac{1}{\cos\frac{\pi}{N_1}} \text{ approx.} \quad (13)$$

In the same way, we get for a 'Humber' chain,

$$\frac{\text{maximum speed}}{\text{minimum speed}} = \frac{1}{\cos a\,O\,b} \text{ (fig. 444)} = \frac{R_1}{\sqrt{R_1^2 - 0{\cdot}3^2}}. \qquad (14)$$

Table XVI. is calculated from formulæ (13) and (14).

TABLE XVI.—VARIATION OF SPEED OF CRANK-AXLE.

Assuming that centres are far apart, and that the number of teeth on chain-wheel of crank-axle is great.

Number of Teeth on Hub		6	7	8	9	10	11	12
Ratio of maximum to minimum speed	Humber .	1·052	1·038	1·028	1·022	1·018	1·015	1·012
	Long-link.	1·155	1·110	1·082	1·064	1·051	1·042	1·035

Discarding the assumptions made above, when N_2 is much greater than N_1, the maximum intercept $O\,e$ (fig. 459) may be appreciably greater than R_1, while the minimum intercept *may* not be appreciably greater than r_1. The variation of the speed-ratio *may* therefore be appreciably greater than the values given in Table XVI.

An important case of chain-gearing is that in which the two chain-wheels are equal, as occurs in tandems, triplets, quadruplets, &c. Drawing figure 459 for this case, it will be noticed that if the distance between the wheel centres be an exact multiple of the pitch of the chain, the lines $O_1\,A$ and $O_2\,C$ are always parallel, the intercept $O_1\,e$ always coincides with $O_1\,A$, and therefore the speed-ratio is constant. If, however, the distance between

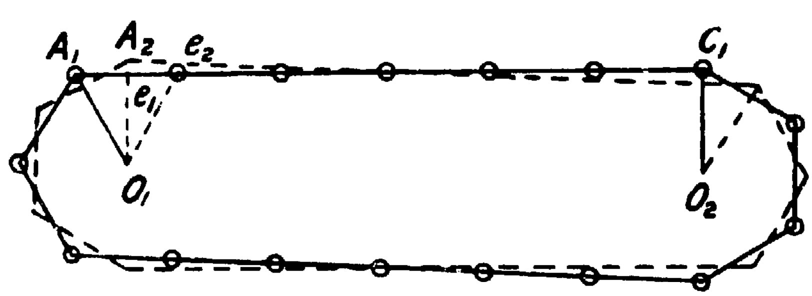


FIG. 460.

the wheel centres be $(k + \frac{1}{2})$ times the pitch, k being any whole number, the variation may be considerable.

In this case, the minimum intercept $O_1\,e_1$ (fig. 460), the

radius $O_1 A_1$ and the chain line $A_1 C_1$ form a triangle $O_1 A_1 e_1$, which is very nearly right-angled at e_1. Therefore for a long-link chain,

$$\frac{\text{minimum speed}}{\text{mean speed}} = \frac{O_1 e_1}{O_1 A_1} = \cos\frac{\pi}{N} \text{ approx.} \quad . \quad . \quad . \quad (15)$$

The corresponding triangle $O_1 A_2 e_2$ formed by the maximum intercept is very nearly right-angled at A_2. Therefore,

$$\frac{\text{maximum speed}}{\text{mean speed}} = \frac{O_1 e_2}{O_1 A_2} = \frac{1}{\cos\frac{\pi}{N}} \text{ approx.} \quad . \quad . \quad . \quad (16)$$

$$\frac{\text{maximum speed}}{\text{minimum speed}} = \frac{1}{\cos^2\frac{\pi}{N}} \text{ approx.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

For a 'Humber' chain,

$$\frac{\text{maximum speed}}{\text{minimum speed}} = \frac{R^2}{R^2 - 0{\cdot}3^2} \text{ approx.} \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Table XVII. is calculated from formulæ (17) and (18).

TABLE XVII.—GREATEST POSSIBLE VARIATION OF SPEED-RATIO OF TWO SHAFTS GEARED LEVEL.

Number of Teeth		8	10	12	16	20	30
Ratio of maximum to minimum speed-ratio . .	Humber chain .	1·058	1·036	1·025	1·014	1·009	1·004
	Long-link chain .	1·190	1·105	1·072	1·039	1·025	1·011

The figures in Table XVI. show that the variation of speed, when a small chain-wheel is used on the driving-hub, is not small enough to be entirely lost sight of. The 'Humber' chain is better in this respect than the long-link chain.

Again, in tandems the speed-ratio of the front crank-axle and the driving-wheel hub is the product of two ratios. The ratio of the maximum to the minimum speed of the front axle *may* be as great as given by the product of the two suitable numbers from Tables XVI. and XVII.

Example.—With nine teeth on the driving-hub, and the two

axles geared by chain-wheels having twelve teeth each, the maximum speed of the front axle *may* be

$$1{\cdot}064 \times 1{\cdot}072 = 1{\cdot}14 \text{ times its minimum speed,}$$

with long-link chains ; and

$$1{\cdot}022 \times 1{\cdot}025 = 1{\cdot}047 \text{ times}$$

with 'Humber' chains.

With triplets and quadruplets the variation *may* be still greater ; and it is open to discussion whether the crank-axles should not be fixed, without chain-tightening gear, at a distance apart equal to some exact multiple of the pitch.

If a hypothetical point be supposed to move with a uniform speed exactly equal to the average speed of a corresponding point actually on the pitch-line of the crank-axle chain-wheel, the distance at any instant between the two is never very great. Suppose the *maximum* speed of the actual point be maintained for a travel of half the pitch, and that it then travels the same distance with its minimum speed. For a speed variation of one per cent. the hypothetical point will be alternately $\frac{1}{800}$th of an inch before and behind the actual point during each inch of travel. This small displacement, occurring so frequently, is of the nature of a vibratory motion, superimposed on the uniform circular motion.

295. **Size of Chain-wheels.**—The preceding section shows that the motion of the crank-axle is more nearly uniform the greater the number of teeth in the chain-wheels. Also, if the ratio of the numbers of teeth in the two wheels be constant, the larger the chain-wheel the smaller will be the pull on the chain. Instead of having seven or eight teeth on the back-hub chain-wheel it would be much better, from all points of view, to have at least nine or ten, especially in tandem machines.

296. **Spring Chain-wheel.**—Any sudden alteration of speed, that is, jerkiness of motion, is directly a waste of energy, since bodies of sensible masses cannot have their speeds increased by a finite amount in a very short interval of time without the application of a comparatively large force. The chain-wheel on the crank-axle revolving with variable speed, if the crank be rigidly connected the pedals will also rotate with variable speed. In the cycle spring chain-wheel (fig. 461) a spring is interposed between the wheel and

the cranks. If, as its inventors and several well-known bicycle manufacturers claim, the wheel gives better results than the ordinary construction, it may be possibly due to the fact that the

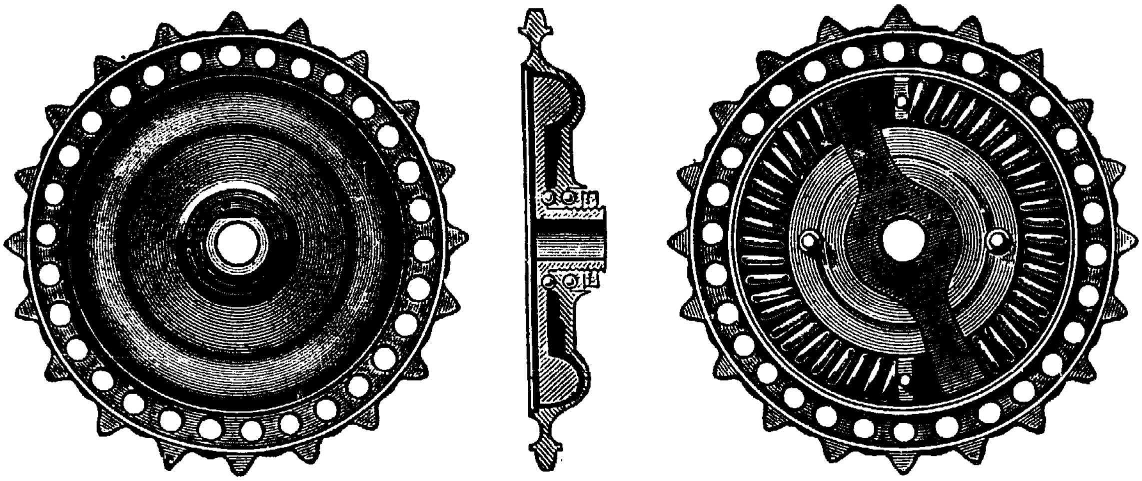


FIG. 461.

spring absorbs as soon as possible the variations of speed due to the chain-driving mechanism, and does not allow it to be transmitted to the pedals and the rider's feet.

If direct spokes are used for the driving-wheel they act as a flexible connection between the hub and rim, allowing the former to run with variable, the latter with uniform speed.

297. **Elliptical Chain-wheel.**—An elliptical chain-wheel has been used on the crank-axle, the object aimed at being an increased speed to the pedals when passing their top and bottom positions, and a diminution of the speed when the cranks are passing their horizontal positions. The pitch-polygon of the chain-wheel in this case is inscribed in an ellipse, the minor axis of which is in line with the cranks (fig. 462).

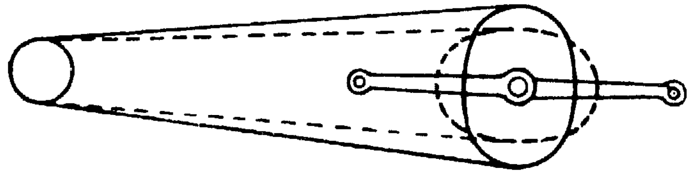


FIG. 462.

The angular speed of the driving-wheel of the cycle being constant and equal to ω, that of the crank-axle is approximately

$$\frac{r_1 \omega}{r_2}$$

where r_1 and r_2 are the radii from the wheel centres to the ends of the straight portion of the chain. r_1 and ω being constant, the angular speed of the crank is therefore inversely proportional to the radius from the centre of the crank-axle to the point at which the driving side of the chain touches the chain-wheel. The speed of the pedals will therefore be least when the cranks are horizontal, and greatest when the cranks are vertical, as indicated by the dotted lines (fig. 462).

If both sides of the chain connecting the two wheels be straight, the total length of the chain as indicated by the full lines (fig. 462) is greater than that indicated by the dotted lines, the difference being due to the difference of the obliquities of the straight portions when the cranks are vertical and horizontal respectively. This difference is very small, and may be practically left out of account. If the wheel centres are very far apart, so that the top and bottom sides of the chain may be considered parallel, the length in contact with the elliptical chain-wheel in any position is evidently equal to half the circumference of the ellipse; similarly, the length in contact with the chain-wheel on the hub is half its circumference, and the length of the straight portions is approximately equal to twice the distance between the wheel centres. Thus the total length is approximately the same, whatever be the position of the chain-wheel.

A pair of elliptical toothed-wheels are sometimes used to connect two parallel shafts. The teeth of these wheels are all of different shapes; there can be at most four teeth in each wheel of exactly the same outline. It has therefore been rather hastily assumed that the teeth of an elliptical chain-wheel must all be of different shapes; but a consideration of the method of designing the chain-wheel (sec. 289) will show that this is not necessarily the case. The investigation there given is applicable to elliptical chain-wheels, and therefore all the teeth may be made from a single milling-cutter, though the form of the spaces will vary from tooth to tooth.

298. **Friction of Chain Gearing.**—There is loss by friction due to the rubbing of the links on the teeth, as they move into, and out of, contact with the chain-wheel. We have seen (sec. 286) that the extent of this rubbing depends on the difference of

the pitches of the chain and wheel; if these pitches be exactly equal, and the tooth form be properly designed, theoretically there is no rubbing. If, however, the tooth outline fall at or near the curve $f\,X$ (fig. 440), the rubbing on the teeth may be the largest item in the frictional resistance of the gearing.

As a link moves into, and out of, contact with the chain-wheel, it turns through a small angle relative to the adjacent link, there is therefore rubbing of the rivet-pin on its bush. While the pin A (fig. 459) moves from the point b_2 to the point a_2 the link $A\,B$ turns through an angle of $\frac{360°}{N_1}$, and the link AA^1 moves practically parallel to itself. The relative angular motion of the adjacent links, BA and AA^1, and therefore also the angle of rubbing of the pin A on its bush, is the same as that turned through by the wheel O_1. In the same way, while the pin D moves from d_1 to c_1 the relative angular motion of the adjacent links $D\,C$ and $C\,C^1$ is the same as the angle turned through by the wheel O_2, viz. $\frac{360°}{N_2}$. The pressure on the pins A and C during the motion is equal to P, the pull of the chain. On the slack side of the chain the motion is exactly similar, but takes place with no pressure between the pins and their bushes. Therefore, the frictional resistance due to the rubbing of the pins on their bushes is the sum of that of two shafts each of the same diameter as the rivet-pins, turning at the same speeds as the crank-axle and driving-hub respectively, when subjected to no load and to a load equal to the pull of the chain.

299. **Gear-Case.**—From the above discussion it will be seen that the chain of a cycle is subjected to very severe stresses, and in order that it may work satisfactorily and wear fairly well it must be kept in good condition. The tremendous bearing pressure on the rivets necessitates, for the efficient working of the chain, constant lubrication. Again, the bending and shearing stresses on the rivets, sufficiently great during the normal working of the chain, will be greatly increased should any grit get between the chain and the teeth of the chain-wheel, and stretching of the chain will be produced. Any method of keeping the chain constantly lubricated and preserving it from dust and grit should add

to the general efficiency of the machine. The gear-case introduced by Mr. Harrison Carter fulfils these requirements. The Carter gear-case is oil-tight, and the chain at its lowest point dips into a small pool of oil, so that the lubrication of the chain is always perfect. The stretching of the chain is not so great with, as without a gear-case ; in fact, some makers go the length of saying that with a Carter gear-case, and the chain properly adjusted initially, there is no necessity for a chain-tightening gear. A great variety of gear-cases have recently been put on the market ; they may be subdivided into two classes : (1) Oil-tight gear-cases, in which the chain works in a bath of oil ; and (2) Gear-cases which are not oil-tight, and which therefore serve merely as a protection against grit and mud. A gear-case of the second type is probably much better than none at all, as the chain, being kept comparatively free from grit, will probably not be stretched so much as would be the case if no gear-case were used.

300. **Comparison of Different Forms of Chain.**—The 'Roller' has the advantage over the 'Humber,' or block chain, that its rubbing surface is very much larger, and that the shape of the rubbing surface—the roller—is maintained even after excessive wear. The 'Roller,' or long-pitched chain, on the other hand, gives a larger variation of speed-ratio than the 'Humber,' or short-pitched chain, the number of teeth in the chain-wheels being the same in both cases ; but a more serious defect of the 'Roller' chain is the imperfect design of the side-plates (sec. 291). If, however, the side-plates of a 'Roller' chain be properly designed, there should be no difficulty in making them sufficiently strong to maintain their shape under the ordinary working stresses. Undoubtedly the weakest part of a cycle chain, as hitherto made, is the rivet, the bending of the rivets probably accounting for most of the stretch of an otherwise well-designed chain. A slight increase in the diameter of the rivets would enormously increase their strength, and slightly increase their bearing surface.

In the 'Humber' there are twice as many rivets as in a 'Roller' chain of the same length. It would probably be improved by increasing the length of the block, until the distance between the centres of the holes was the same as between the holes in the side-plates. This would increase the pitch to 1·2 inches, without

in any way increasing the variation of the speed-ratio. In order to still further reduce the number of rivets, the pitch might be

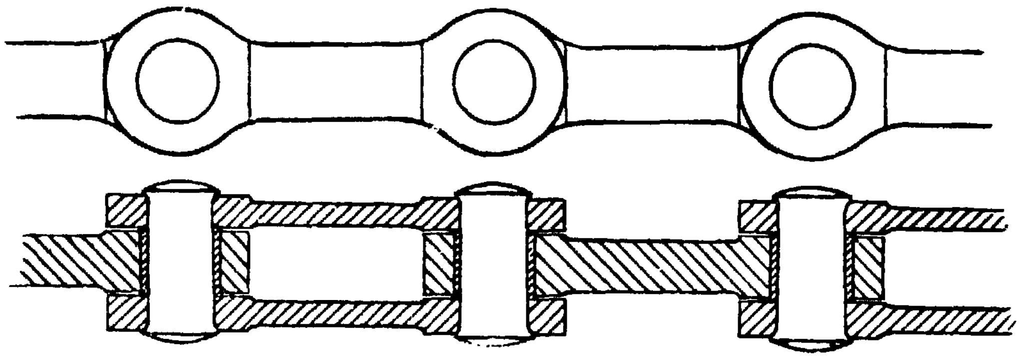


FIG. 463.

increased to one inch, giving a total pitch of two inches for the chain. If the side-plates be made to the same outline as the middle block (fig. 463), they may also be used to come in contact with the teeth of the chain-wheel. The chain-wheel would then have the form shown in figure 464, in which the alternate teeth are in duplicate at the edges of the rim. For tandems, triplets, &c., a still greater pitch, say $1\frac{1}{2}$ inches, may be used with advantage; the back-hub chain-wheel, with six teeth of this pitch, would have the same average radius of pitch-polygon as a chain-wheel with nine teeth of 1 inch pitch; the chain would have only one-third the number of rivets in an ordinary 'Humber' chain of the same length, and if the rivets were made slightly larger than usual, *stretching* of the chain might be reduced to zero.

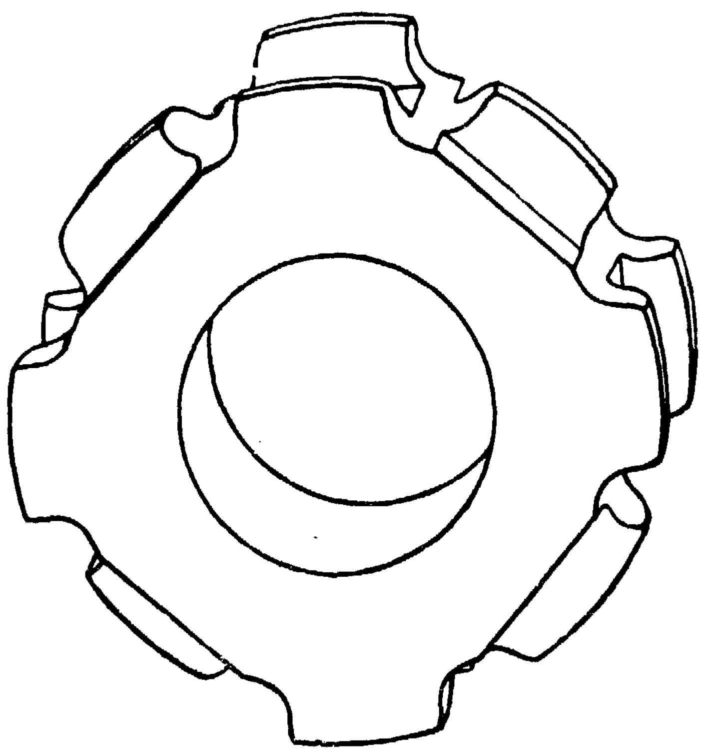


FIG. 464.

301. **Chain-tightening Gear.**—The usual method of providing for the chain adjustment is to have the back-hub spindle fastened

to a slot in the frame, the length of slot being at least equal to half the pitch of the chain. In the swinging seat-strut adjustment, the slot is made in the lower back fork, and the lower ends of the seat-struts are provided with circular holes through which the spindle passes. These have been described in the chapter on Frames.

The 'eccentric' adjustment is almost invariably used for the front chain of a tandem bicycle. The front crank-axle is carried on a block, the outer surface of which is cylindrical and eccentric to the centre of the axle. The adjustment is effected by turning

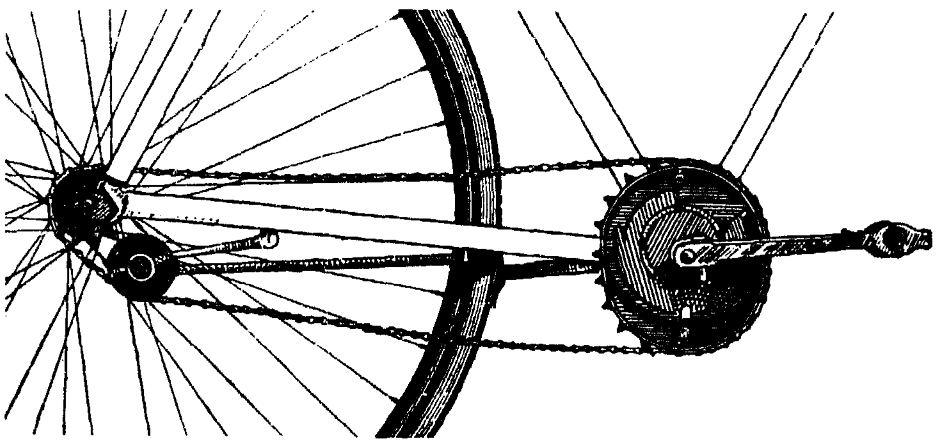


FIG. 465.

the block in the bottom-bracket, and clamping it in the desired position.

A loose pulley carried at the end of a rod controlled by a spring (fig. 465) is used in conjunction with Linley & Biggs' expanding chain-wheel.

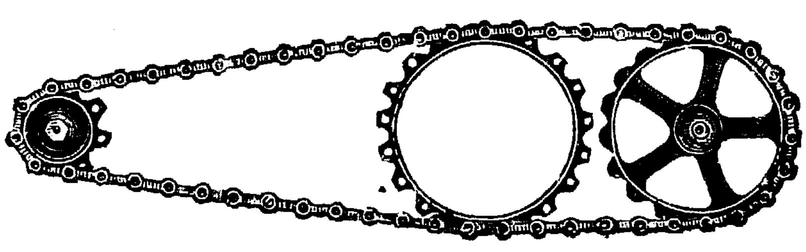


FIG. 466.

Figure 466 illustrates a method used at one time by Messrs. Hobart, Bird & Co. When the chain required to be tightened, the loose chain-wheel was placed nearer the hub chain-wheel.

CHAPTER XXVII

TOOTHED-WHEEL GEARING

302. **Transmission by Smooth Rollers.**—Before beginning the study of the motion of toothed-wheels, it will be convenient to take that of wheels rolling together with frictional contact; since a properly designed toothed-wheel is kinematically equivalent to a smooth roller.

Parallel Shafts.—Let two cylindrical rollers be keyed to the shafts A and B (fig. 467); if one shaft revolves it will drive the other, provided the frictional resistance at the point of contact of the rollers is great enough to prevent slipping. When there is no slipping, the linear speeds of two points, one on the circumference of each roller, must be the same. Let ω_1 and ω_2 be the angular speeds of the shafts, r_1 and r_2 the radii of the rollers; then the above condition gives

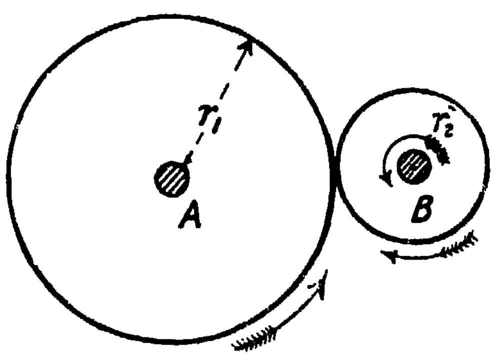


FIG. 467.

$$\omega_1 r_1 = - \omega_2 r_2,$$

or

$$\frac{\omega_1}{\omega_2} = - \frac{r_2}{r_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

the negative sign indicating that the shafts turn in opposite directions. Thus the angular speeds are inversely proportional to the radii (or diameters) of the rollers.

If the smaller roller lie inside the larger, they are said to have internal contact, and the shafts revolve in the same direction.

Intersecting Shafts.—Two shafts, the axes of which intersect,

may be geared together by conical rollers, the vertices of the two cones coinciding with the point of intersection of the shafts. Figure 468 shows diagrammatically two shafts at right angles, geared together by rollers forming short frusta of cones. If there be no slipping at the point of contact, the linear speeds of two points, situated one on each wheel, which touch each other during contact, must be equal. Equation (1) will hold in this case, r_1 and r_2 being the radii of the bases of the cones.

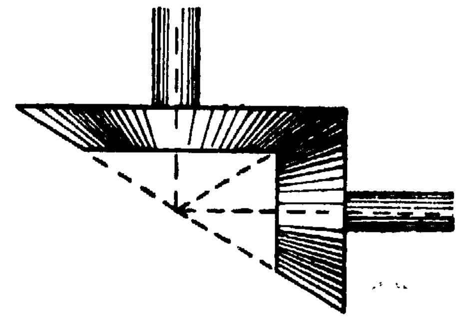


FIG. 468.

Two shafts whose axes are not parallel and do not intersect may be connected by rollers, the surfaces of which are hyperboloids of revolution. The relative motion will, however, not be pure rolling, but there will be a sliding motion along the line of contact of the rollers, which will be a generating straight line of each of the hyperboloids. This form of gear, or its equivalent hyperboloid skew-bevel gear, has not been used to any great extent in cycle construction, and will therefore not be discussed in the present work.

303. **Friction Gearing.**—If two smooth rollers of the form above described be pressed together there will be a certain frictional resistance to the slipping of one on the other, and hence if one shaft is a driver the other may be driven, provided its resistance to motion is less than the frictional resistance at the surface of the roller. Friction rollers are used in cases where small driving efforts have to be transmitted, but when the driving effort is large, the necessary pressure between the rollers would be so great as to be very inconvenient. In 'wedge gearing,' the surfaces are made so that a projection of wedge section on one roller fits into a corresponding groove on the other; the frictional resistance, for a given pressure, being thereby greatly increased.

304. **Toothed-wheels.**—When the effort to be transmitted is too large for friction gearing to be used, projections are made on one wheel and spaces on the other; a pair of toothed-wheels are thus obtained.

Toothed-wheels should have their teeth formed in such a manner that the relative motion is the same as that of a pair

of toothless rollers. The surfaces of the equivalent toothless rollers are called the *pitch surfaces* of the wheels. By the radius or diameter of a toothed-wheel is usually meant that of its pitch surface; equation (1) will therefore be true for toothed-wheels. The distance between the middle points of two consecutive teeth measured round the pitch surface is called the *pitch* or the *circular pitch* of the teeth. The pitch must evidently be the same for two wheels in gear. Let p be the pitch, N_1 and N_2 the numbers of teeth in the two wheels, and n_1 and n_2 the numbers of revolutions made per minute; then the spaces described by two points, one on each pitch surface, in one minute are equal; therefore

$$2\,\pi\,n_1\,r_1 = 2\,\pi\,n_2\,r_2.$$

Since

$2\,\pi\,r_1 = N_1\,p$, and $2\,\pi\,r_2 = N_2\,p$, we get

$$N_1\,n_1\,p = N_2\,n_2\,p$$

or,

$$\frac{N_1}{N_2} = \frac{n_2}{n_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

That is, the angular speeds of the toothed-wheels in contact are inversely proportional to the numbers of teeth.

If the pitch diameter be a whole number, the circular pitch will be an incommensurable number. The *diametral pitch* is defined by Professor Unwin as "A length which is the same

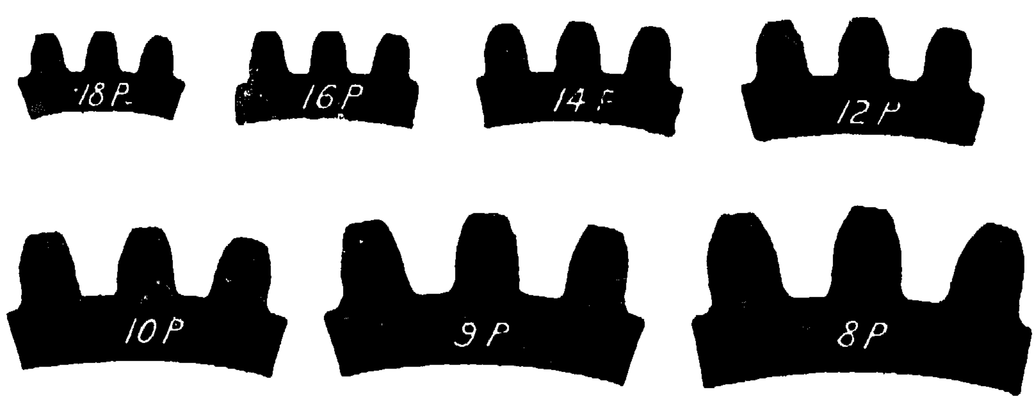


FIG. 469.

fraction of the diameter as the circular pitch is of the circumference." The American gear-wheel makers define the diametral pitch as "The number of teeth in the gear divided by the pitch diameter of the gear." The latter may be called the *pitch-number*.

It is much more convenient to use the pitch-number than the circular pitch to express the size of wheel-teeth. Figure 469 shows the actual sizes of a few teeth, with pitch-numbers suitable for use in cycle-making. If p be the circular pitch, s the diametral pitch, and P the pitch-number,

$$s = \frac{p}{\pi}$$

$$P = \frac{1}{s} = \frac{\pi}{p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

305. **Train of Wheels.**—If the speed-ratio of two shafts to be geared together by wheels be large, to connect them by a single pair of wheels will be in most cases inconvenient; one wheel of the pair will be very large and the other very small. In such a case one or more intermediate shafts are introduced, so that the speed-ratio of any pair of wheels in contact is not very great. The whole system is then called a *train* of wheels. For example, in a watch the minute hand makes one complete revolution in one hour, the seconds hand in one minute; the speed-ratio of the two spindles is 60 to 1; here intermediate spindles are necessary.

If the two shafts to be connected are coaxial, it is *kinematically necessary*, not merely convenient, to employ a train of wheels. This is the case of a wheel or pulley rotating loosely on a shaft, the two being geared to have different speeds. Figure 470 shows the simplest form of gearing of this description, universally used to form the slow gearing of lathes, and which has been extensively used to form gears for front-driving Safeties. A is the shaft to which is rigidly fixed the wheel D, gearing with the wheel E on the intermediate shaft B. The bearings of the shafts A and B are carried by the frame C. On the shaft B is fixed another wheel F, gearing with the wheel G, rotating loosely on the shaft A.

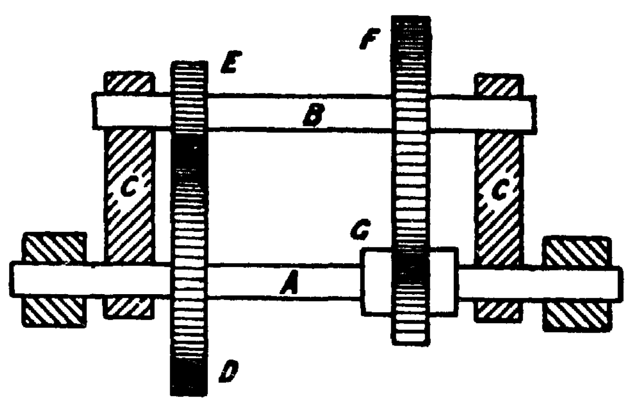


FIG. 470.

Denoting the number of teeth in a wheel by the corresponding

small letter, the speed-ratio of the shafts B and A will be $-\frac{d}{e}$; the negative sign indicating that the shafts turn in opposite directions. The speed-ratio of the wheels G and F will be $-\frac{f}{g}$, and the speed-ratio of the wheels G and D will be the product

$$\frac{d}{e} \cdot \frac{f}{g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The wheels D and G (fig. 470) revolve in the same direction, the four wheels in the gear all having external contact. If one of the pairs of wheels has internal contact, the wheels A and G will revolve in opposite directions. The speed-ratio will then be

$$-\frac{d}{e} \cdot \frac{f}{g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

306. **Epicyclic Train.**—The mechanism (fig. 470) may be inverted by fixing one of the wheels D or G and letting the frame-link C revolve; such an arrangement is called an *epicyclic train.* The speed-ratio of the wheels D and G relative to C will still be expressed by (4). Suppose D the wheel fixed, also let its angular speed relative to the frame-link C be denoted by unity, and that of G by n. When the frame-link C is at rest its angular speed about the centre A is zero. The angular speeds of D, C, and G are then proportional to 1, 0, and n. Let an angular speed -1 be added to the whole system; the angular speeds of D, C, and G will then be respectively

$$0, \; -1, \text{ and } n-1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If one pair of wheels has internal contact, the angular speeds of D, C, and G will be represented by -1, 0, and n; adding a speed $+1$ to the system, the speeds will become respectively

$$0, \; 1, \text{ and } n+1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

An epicyclic train can be formed with four bevel-wheels (fig. 471); also, instead of two wheels, E and F (fig. 470), only one may be used which will touch A externally and G internally (fig.

472); this is the kinematic arrangement of the well-known 'Crypto' gear for Front-drivers. Again, in a bevel-wheel epicyclic train the two wheels on the intermediate shaft B may be merged into one; this is the kinematic arrangement of Starley's balance gear for tricycle axles (fig. 219).

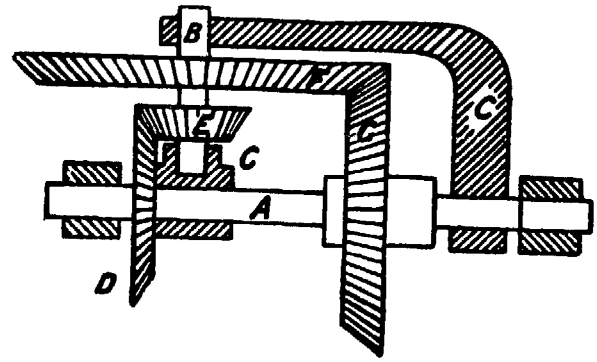


FIG. 471.

In a Crypto gear, let N_1 and N_2 be the numbers of teeth on the hub wheel and the fixed wheel respectively, then $n = \frac{N_2}{N_1}$; and from (6), the speed-ratio of the hub and crank is

$$\frac{N_2}{N_1} + 1 = \frac{N_1 + N_2}{N_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

From (7) it is evident that if a speed-ratio greater than 2 be desired, N_2 must be greater than N_1, and the annular wheel must therefore be fixed to the frame and the inner wheel be fixed to the hub.

Example.—The fixed wheel D of a Crypto gear has 14 teeth, the wheel E mounted on the arm C has 12 teeth; the number of

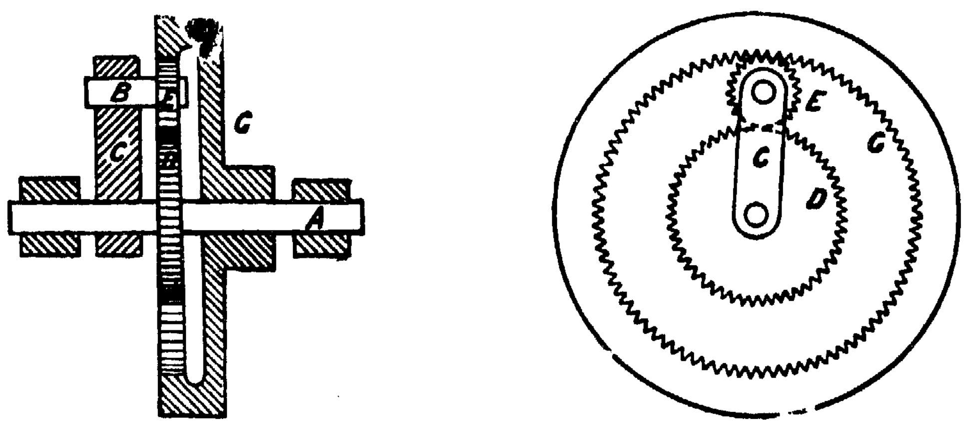


FIG. 472.

teeth in the wheel G fixed to the hub of the driving-wheel must then be $12 + 12 + 14 = 38$. The driving-wheel of the bicycle is 46 in. diameter; what is it geared to? Substituting in (7), the speed-ratio of the hub and the crank is $\frac{52}{38}$, and the bicycle is geared to $\frac{52 \times 46}{38} = 62{\cdot}95$ inches.

307. **Teeth of Wheels.**—The projection of the pitch-surface of a toothed-wheel on a plane at right angles to its axis is called the *pitch-circle*; a concentric circle passing through the points of the teeth is called the *addendum-circle*; and a circle passing through the bases of the teeth is called the *root-circle* (fig. 473). The part of the tooth surface *b c* outside the pitch-line is called the *face*, and the part *a b* inside the pitch-circle the *flank* of

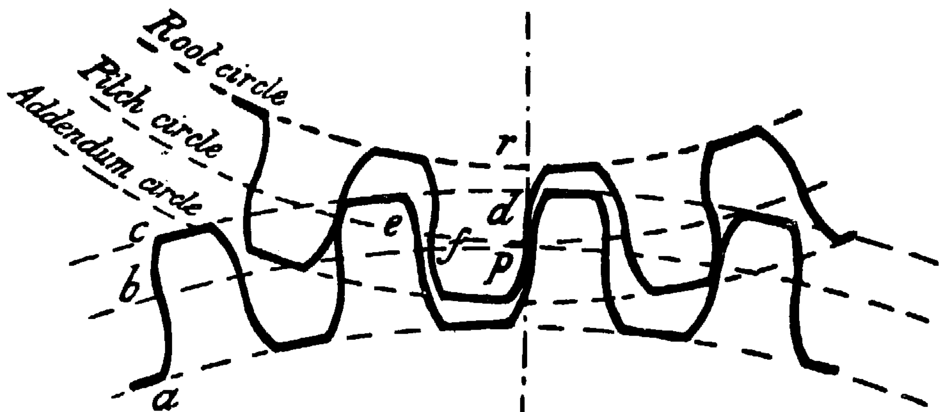


FIG. 473.

the tooth. The portion of the tooth outside the pitch-circle is called the *point*; and the portion inside, the *root*. The line joining the wheel centres is called the *line of centres*. The *top and bottom clearance* is the distance *r d* measured on the line of centres, between the addendum-circle of one wheel and the root-circle of the other. The *side-clearance* is the difference *e f* between the pitch and the sum of the thicknesses of the teeth of the two wheels, measured on the pitch-circle.

For the successful working of toothed-wheels forming part of the driving mechanism of cycles it is absolutely necessary not only that the tooth forms should be properly designed, but also that they be accurately formed to the required shape. This can only be done by cutting the teeth in a special wheel-cutting machine. In these machines, the milling-cutter being made initially of the proper form, all the teeth of a wheel are cut to exactly the same shape, and the distances measured along the pitch-line between consecutive teeth are exactly equal. In slowly running gear teeth, as in the bevel-wheels in the balance gear of a tricycle axle, the necessity for accurate workmanship is not so great, and the teeth of the wheels may be cast.

308. **Relative Motion of Toothed-wheels.**—Let $a\,F\,a$ and $b\,F\,b$ (fig. 474) be the outlines of the teeth of wheels, F being the point of contact of the two teeth. Let D be the centre of curvature of the portion of the curve $a\,F\,a$ which lies very close to the point F; that is, D is the centre of a circular arc approximating very closely to a short portion of the curve $a\,F\,a$ in the neighbourhood of the point F. Similarly, let C be the centre of curvature of the portion of the curve $b\,F\,b$ near the point F. Whatever be the tooth-forms $a\,F\,a$ and $b\,F\,b$, it will in general be possible to find the points D and C, but the positions of C and D on the respective wheels change as the wheels rotate and the point of contact F of the teeth changes. While the wheels A and B rotate through a small angle near the position shown, their motion is exactly the same as if the points C and D on the wheels were connected by a link $C\,D$. The instantaneous motion of the two wheels is thus reduced to that of the levers $A\,C$ and $B\,D$ connected by the coupler $C\,D$.

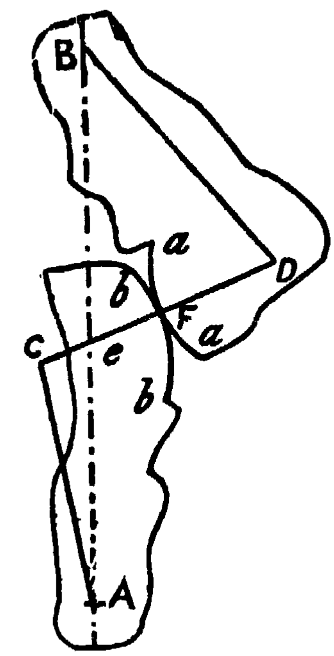


FIG. 474.

In figure 21 (sec. 32) let $B\,A$ and $C\,D$ be produced to meet at J; then

$$\frac{D\,e}{C\,B} = \frac{D\,J}{C\,J}, \text{ or } D\,e = \frac{D\,J}{C\,J} \cdot C\,B.$$

And it has been already shown that the speed-ratio of the two cranks is $\frac{D\,e}{D\,A}$. Therefore the speed-ratio may be written equal to $\frac{C\,B}{D\,A} \cdot \frac{D\,J}{C\,J}$.

Therefore, since $C\,B$ and $D\,A$ are constant whatever be the position of the mechanism (fig. 21), the angular speeds of the two cranks in a four-link mechanism are inversely proportional to the segments into which the line of centres is divided by the centre-line of the coupling-link. Therefore if the straight line $C\,D$ cut $A\,B$ at e (fig. 474) the speed-ratio of the wheels A and B is

$$\frac{B\,e}{A\,e} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

For toothed-wheels to work smoothly together the angular speed-ratio should remain constant ; (8) is therefore equivalent to the following condition : *The common normal to a pair of teeth at their point of contact must always pass through a fixed point on the line of centres.* This fixed point is called the *pitch-point*, and is evidently the point at which the pitch-circles cut the line of centres.

If the form of the teeth of one wheel be given, that of the teeth of the other wheel can in general be found, so that the above condition is satisfied. This problem occurs in actual designing when one wheel of a pair has been much worn and has to be replaced. But in designing new wheels it is of course most convenient to have the tooth forms of both wheels of the same general character. The only curves satisfying this condition are those of the trochoid family, of which the cycloid and involute are most commonly used.

309. **Involute Teeth.**—Suppose two smooth wheels to rotate about the centres A and B (fig. 475), the sum of the radii being less than the distance between the centres. Let a very thin cord be partially wrapped round one wheel, led on to the second wheel, and partially wrapped round it. Let c and d respectively be the points at which the cord leaves A and touches the wheel B. Let a pencil, P, be fixed to the cord, and imagine a sheet of paper fixed to each wheel. Then the cord not being allowed to slip round either wheel, while the point P of the string moves from c to d, the wheels A and B will be driven, and the pencil will trace out on the paper fixed to A an arc of an involute $a\ a^1$, and on the paper fixed to B an involute arc $b^1\ b$. If teeth-outlines be made to these curves they must touch each other at some point on the

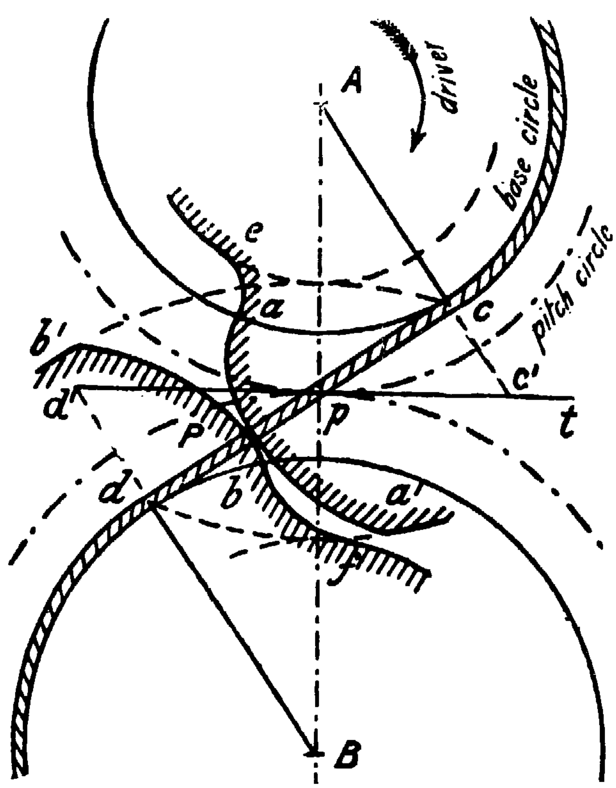


FIG. 475.

line $c\,d$, which is their common normal at their point of contact; and since $c\,d$ intersects the line of centres at a fixed point, p, the tooth-outlines satisfy the condition for constant speed-ratio.

The circles round which the cord was supposed to be wrapped are called the *base-circles* of the involute teeth; the line $c\,d$ is called the *path of the point of contact*, or simply the *path of contact*. The longest possible involute teeth are got by taking the addendum-circles of wheels A and B through d and c respectively; for though the involutes may be carried on indefinitely outwards from the base-circles, no portion can lie inside the base-circle. Except in wheels having small numbers of teeth, the arcs of the involutes used to form the tooth outlines are much smaller than shown in figure 475; the path of contact being only a portion of the common tangent $c\,d$ to the base-circles.

The *angle of obliquity* of action is the angle between the normal to the teeth at their point of contact, and the common tangent at p to the pitch-circles. The angle of obliquity of involute teeth is constant, and usually should not be more than 15°.

No portion of a tooth lying inside the base-circle has working contact with the teeth of the other wheel, but in order that the points of the teeth of the wheels may get past the line of centres, the space between two adjacent teeth must be continued inside the base-circle. If the teeth be made with no clearance the continuation of the tooth outline $b^1\,P\,b$, between the base- and root-circles, is an arc of an epitrochoid $b\,f$, described on the wheel B by the point a^1 of the wheel A. The continuation of the tooth outline $a^1\,P\,a$ between the base- and root-circles is an arc of an epitrochoid $a\,e$, described on the wheel A by the point b^1 of the wheel B. This part of the tooth outline lying between the root-circle and the working portion of the tooth outline is sometimes called the *fillet*. The flanks are sometimes continued radially to the root-circle; but where the strength of the teeth is of importance, the fillet should be properly designed as above. The *fillet-circle* is a circle at which the fillets end and the working portions of the teeth begin. When involute teeth are made as long as possible, the base- and fillet-circles coincide. In any case, the fillet-circle of one wheel and the addendum-circle of the other pass through the same point, at the end of the path of contact.

Let the centres A and B of the wheels (fig. 475) be moved farther apart; the teeth will not engage so deeply, and the line dc will make a larger angle with the tangent to the pitch-circles at p. The form of the involutes traced out by the pencil will, however, be exactly the same though a longer portion will be drawn. Therefore the teeth of the wheels will still satisfy the condition of constant speed-ratio. Wheels with involute teeth have therefore the valuable property that the distance between

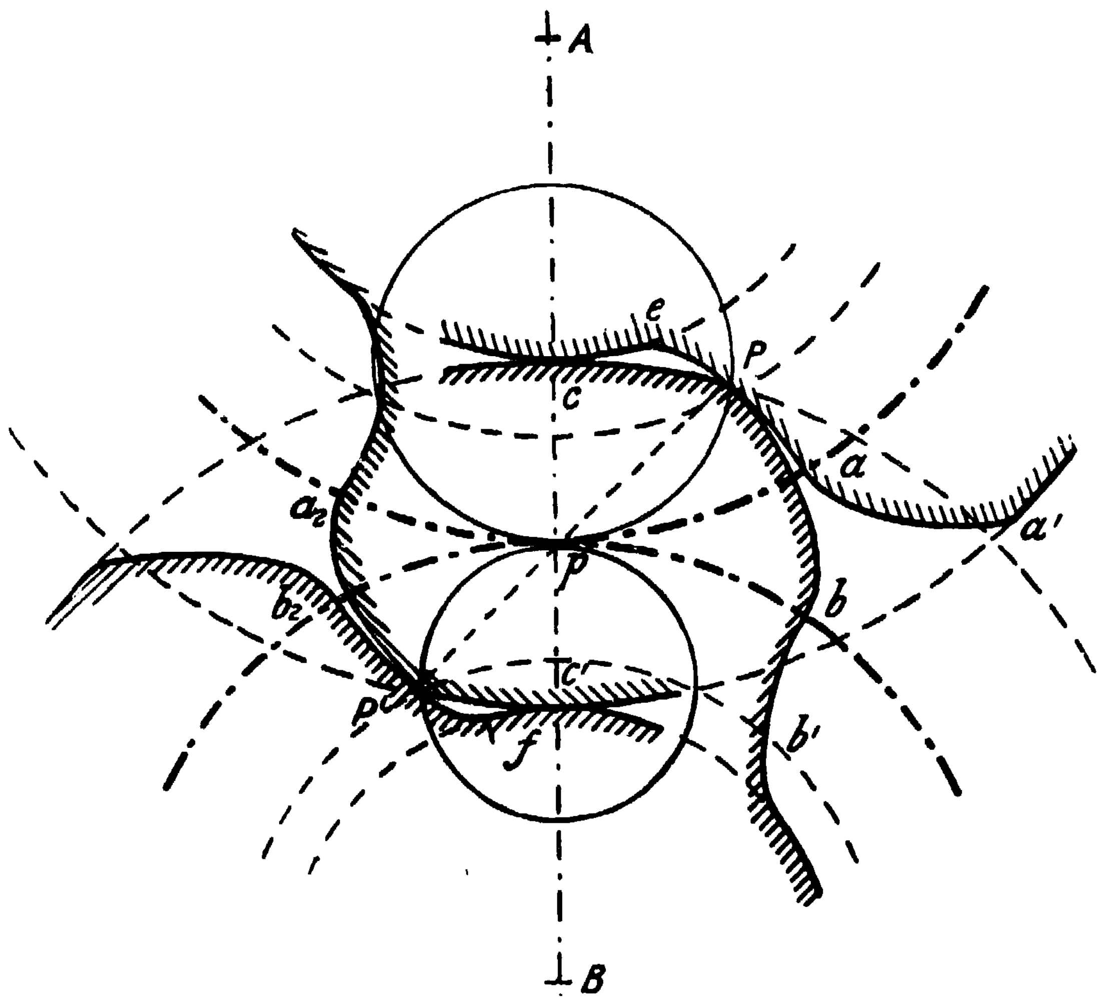


FIG. 476.

their centres may be slightly varied without prejudicially affecting the motion.

310. **Cycloidal Teeth.**—Let A and B (fig. 476) be the centres of two wheels, and let p be the pitch-point. Let a third circle with centre C lying inside the pitch-circle of A roll in contact with the two pitch-circles at the pitch-point. Suppose a pencil P fixed to the circumference of the rolling-circle. If the three circles roll so that p is always their common point of contact, the

pencil will trace out an epicycloid on wheel B, and an hypocycloid on A. Let P be any position of the pencil, then the relative motion of the two circles A and C is evidently the same as if A were fixed and C rolled round inside it; p is therefore the instantaneous centre of rotation of the circle C, and the direction of motion of P relative to the wheel A must be at right angles to the line $p\,P$; that is, $p\,P$ is the normal at P to the hypocycloid $P\,a$.

In the same way it can be shown that the line $p\,P$ is the normal to the epicycloid $P\,b$. If tooth outlines be made to these curves they will evidently satisfy the condition for constant speed-ratio.

The tooth $P\,a$ is all flank, and the tooth $P\,b$ all face. Another rolling-circle C^1 may be taken inside the pitch-circle of wheel B, a tracing-point P^1 on it will describe an epicycloid on wheel A and a hypocycloid on wheel B. The tooth outline $P^1\,a_2$ is all face, and the tooth outline $P^1\,b_2$ is all flank. They may be combined with the former curves, so that the tooth outlines $P\,a\,a^1$ and $P\,b\,b^1$ may be used.

The path of contact $P^1\,p\,P$ in this case is evidently made up of arcs of the two rolling-circles. If the diameter of the rolling-circle be equal to the radius of the pitch-circle, the hypocycloid described reduces to a straight line a diameter of the pitch-circle. The flanks of cycloidal teeth may therefore be made radial.

If contact begins and ends at the points P and P^1 respectively, the addendum-circles of B and A pass through these points. If the teeth are made without clearance, the fillet will be, as in involute teeth, an arc of an epitrochoid, $P\,e$, described on the wheel A by the point P of the wheel B. Similarly, the fillet of wheel B between P^1 and the root-circle is an arc of an epitrochoid $P^1\,f$ described on wheel B by the point P^1 of the wheel A.

If a set of wheels with cycloidal teeth are required, one wheel of the set to gear with any other, the same rolling-circle must be taken for the faces and flanks of all.

An important case of cycloidal teeth is that in which the rolling-circle is equal to the pitch-circle of one of the wheels of the pair. The teeth of one wheel become points; those of the other, epicycloids described by one pitch-circle rolling on the other.

If two tooth outlines gear properly together with constant

speed-ratio, tooth outlines formed by parallel curves will in general also gear together properly. In the above case the *point* teeth of one wheel may be replaced by round *pins*, the epicycloid teeth of the other wheel by a parallel curve at a distance equal to the radius of the pin. Loose rollers are sometimes put round the pins so that the wear is distributed over a larger surface.

An example of pin-gearing is found in the early patterns of the 'Collier' two-speed gear (sec. 319).

311. **Arcs of Approach and Recess.**—The *arc of approach* is the arc through which a point on the pitch-circle moves from the time that a pair of teeth come first into contact until they are in contact at the pitch-point. The *arc of recess* is the arc through which a point on the pitch-circle moves from the time a pair of teeth are in contact at the pitch-point until they go out of contact. The *arc of contact* is, of course, the sum of the arcs of approach and recess.

With cycloidal teeth (fig. 476), if P and P^1 be the points of contact when the teeth are just beginning and just leaving contact respectively, ap or bp will be the arc of approach, and $p\,a_2$ or $p\,b_2$ the arc of recess, provided the wheel A is the driver, in watch-hand direction. From the mode of generation of the epicycloid and the hypocycloid it is evident that the arc of the rolling circle, Pp, is equal to the arc of approach, and $p\,P^1$ to the arc of recess.

With involute teeth (fig. 475) the path of contact is the straight line cd. The arc of contact, measured along either of the base-circles, is equal to cd. The arc of contact, measured along the pitch-circle, is equal to $\overline{cd}$ multiplied by $\frac{Ap}{Ac}$, the ratio of the radii of the pitch- and base-circles. Draw the tangent pt at p to the pitch-circles, and produce Ac and Bd to meet pt at c^1 and d^1 respectively. c^1p and pd^1 are the lengths of the arcs of approach and recess respectively, measured along the pitch-circle. For from similar triangles,

$$\frac{pc^1}{pc} = \frac{Ap}{Ac}, \text{ or } pc^1 = \frac{Ap}{Ac}\cdot pc.$$

Similarly $$pd^1 = \frac{Ap}{Ac}\cdot pd.$$

312. **Friction of Toothed-wheels.**—There is a widespread impression, even among engineers, that, if the form of wheel-teeth be correctly designed, the relative motion of the teeth is one of pure rolling. Probably the use of the term *rolling-circles* in connection with cycloidal teeth has given rise to this impression; but a very slight inspection of figures 475 and 476 will show that the teeth rub as well as roll on each other. In figure 476 a pair of teeth are shown in contact at P. While the teeth are passing the pitch-point, the points a and b touch each other at p. Now the length Pa of one tooth is much less than the length Pb of the other. The teeth must therefore rub on each other a distance equal to the difference between these two arcs. The same thing is apparent from figure 475.

The speed of rubbing at any point can be easily expressed as follows: Let a pair of wheels rotate about the centres A and B (fig. 477); let their pitch-lines touch at p; let $a'pa''$ be the path of contact; let r_1 and r_2 be the radii of the pitch-circles, and V be their common linear speed, the angular speeds will be $\frac{V}{r_1}$ and $-\frac{V}{r_2}$ respectively. Suppose a pair of teeth to be in contact at a; the relative motion of the two wheels will be the same if the whole system be given a rotation $\frac{V}{r_2}$ about B in the direction opposite to the rotation of the wheel B. Wheel B will now be at rest, and the pitch-line of wheel A will roll on the pitch-line of B. The angular speed of wheel A is now $\frac{V}{r_1}+\frac{V}{r_2}$. The instantaneous centre of rotation of wheel A is the point p, and therefore the linear speed of the point a on the wheel A is

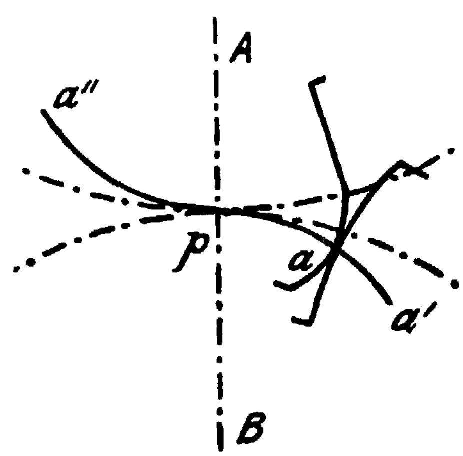


FIG. 477.

$$V\left(\frac{1}{r_1}+\frac{1}{r_2}\right) \times \text{chord } pa \quad \ldots \ldots \quad (9)$$

This is, of course, the same as the relative speed of rubbing of the teeth in contact at a. In particular, the speed of rubbing is

greatest when the teeth are just coming into or just leaving contact, and is zero when the teeth are in contact at the pitch-point.

If the two wheels have internal contact, by the same reasoning their relative angular speed may be shown to be $V\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$, and the speed of rubbing

$$V\left(\frac{1}{r_1}-\frac{1}{r_2}\right) \times \text{chord } pa \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Thus, comparing two pairs of wheels with external and internal contact respectively, if the pitch-circles and arc of contact be the same in both, the wheels with internal contact have much less rubbing than those with external contact. If $r_2 = 3r_1$ the rubbing with external contact is twice as great as with internal.

Friction and Wear of Wheel-teeth.—The frictional resistance, and therefore the wear, of wheel-teeth will be proportional to the maximum speed of rubbing, and will therefore be greater the longer the path of contact. The arc of contact, therefore, should be chosen as short as possible; the working length of the teeth will then be short, and it will be much easier to make the teeth accurately. The arc of contact must, of course, be at least equal to the pitch, so that one pair of teeth comes into contact before the preceding pair has left contact. It may be chosen a little greater, in order to allow a margin for the centres of the wheels being moved a little further apart than was intended. The rubbing of the teeth against each other during approach is said to be more injurious than during recess. In a pair of wheels in which the driver and driven are never interchanged (as in gear-wheels of cycles, which are always driven ahead and never backwards), the arc of recess may therefore be chosen a little larger than the arc of approach.

If c be the length of the arc of contact, the *average* speed of rubbing will be approximately (with external contact)

$$\frac{c}{4}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)V.$$

If P be the average normal pressure on the teeth, and μ the coefficient of friction, the work lost in friction will be

$$\frac{c}{4}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\mu P V.$$

The useful work done in the same time will be approximately $P V$, and the efficiency of the gear will be

$$\frac{P V}{P V+\frac{c}{4}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\mu P V}=\frac{1}{1+\frac{c \mu}{4}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)} \quad . \quad . \quad . \quad (11)$$

Example I.—In a pair of wheels with 12 and 24 teeth respectively, assuming $\mu = \cdot 08$, and $c = 1 \cdot 2 p$, $\frac{p}{r_1} = \frac{2\pi}{12} = \cdot 524$, and $\frac{p}{r_2} = \cdot 262$, and the efficiency is

$$\frac{1}{1+\cdot 3 \times \cdot 08\,(\cdot 524+\cdot 262)}=\cdot 982.$$

Example II.—In an internal gear with 12 and 36 teeth, with the same assumptions as above, the efficiency is

$$\frac{1}{1+\cdot 024\,(\cdot 524-\cdot 175)}=\cdot 992.$$

313. **Circular Wheel-teeth.**—Since only a small arc is used to form the tooth outline it is often convenient to approximate to the exact curve by a circular arc. Involute or cycloidal teeth are first designed by the above methods, then the circular arcs, which fit as closely as possible, are used for the actual tooth outlines. When this is done there will be a slight variation of the speed-ratio during the time of contact of a pair of teeth. The variation may be reduced to a minimum by (instead of proceeding as just described) finding the values of $A C$, $B D$, and $C D$ (fig. 474), such that the point e will deviate the smallest possible amount from p, the pitch-point. The author has investigated this subject in a paper on 'Circular Wheel-teeth,' published in the 'Proceedings of the Institution of Civil Engineers,' vol. cxxi. The analysis is too long for insertion here, but the principal results may be given :

For a given value of the speed-ratio $R = \frac{r_2}{r_1}$, three positions of the coupling-link CD can be found in which it passes through the pitch-point p: let $C_1 D_1$, $C_2 D_2$ and $C_3 D_3$ (fig. 478) be these positions. The distance of C_2, the middle position of C,

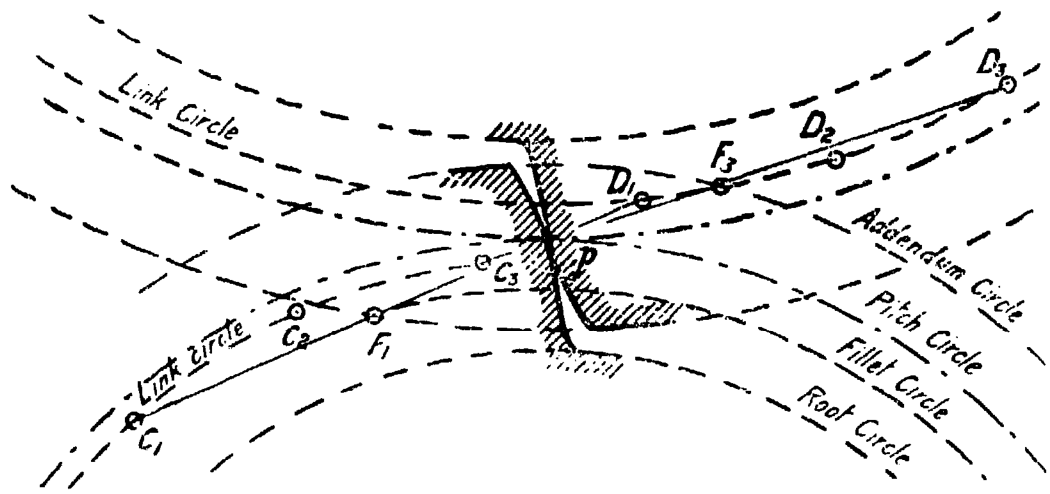


FIG. 478.

from the pitch-point p may be chosen arbitrarily; but the greater this distance the less will be the speed variation and the greater the obliquity.

Let
$$\frac{p\,C_2}{r_1} = m, \quad \frac{\text{arc of contact}}{r_1} = 2\,n\,;$$

then assuming that the other two positions of the equivalent link in which its intersection e with the line of centres coincides with the pitch-point p are at the beginning and end of contact of a pair of teeth, we may take approximately

$$p\,C_1 = (m + n)\,r_1, \quad p\,C_2 = m\,r_1, \quad p\,C_3 = (m - n)\,r_1.$$

Let $A\,C = l_1$, $B\,D = l_2$, and the length $C\,D$ of the equivalent coupling-link $= h$. The values of h, l_1, and l_2, for given values of R, m, and n, are given by the following equations.

$$\frac{h}{r_1} = \frac{3\,(R + 1)}{(R + 2)}\,m \quad \ldots \ldots \quad (12)$$

$$\left(\frac{l_1}{r_1}\right)^2 = 1 - \frac{(R + 2)}{3\,R}\,(m^2 - n^2) \quad \ldots \ldots \quad (13)$$

$$\left(\frac{l_2}{r_2}\right)^2 = 1 - \frac{(2\,R + 1)^3}{3\,R^2\,(R + 2)^2}\,m^2 + \frac{(2\,R + 1)}{3\,R^2}\,n^2 \quad \ldots \quad (14)$$

Also V, the percentage speed variation above and below the average, is given by the equation

$$V = \frac{6 \cdot 415 (R + 1)(R + 2)}{R^2} \cdot \frac{n^3}{m} \quad . \quad . \quad . \quad . \quad . \quad (15)$$

from which, for a constant value of m, the variation is inversely proportional to the cube of the number of teeth in the smaller wheel. The values of $\frac{h}{r_1}$, $\frac{l}{r_1}$, $\frac{l_2}{r_2}$, and V, for $m = \cdot 3$ and various values of R and n, are given in Table XVIII.

Having calculated, or found from the tables, the values of l_1, l_2, and h, the drawing of the teeth may be proceeded with as follows :

Draw the pitch-circles, with centres A and B, touching at the pitch-point p ; draw the link-circle $C_1\ C_2\ C_3$ with centre A and radius l_1 ; likewise draw the link-circle $D_1\ D_2\ D_3$. With centre p and radii equal to $(m + n) r_1$, $m\, r_1$, and $(m - n) r_1$ respectively, draw arcs cutting the link-circle C at the points C_1, C_2, and C_3, respectively. With centres C_1, C_2, and C_3, and radius equal to h, draw arcs cutting the link-circle D at D_1, D_2, and D_3 respectively. A check on the accuracy of the drawing and calculation is got from the fact that the straight lines $C_1\, D_1$, $C_2\, D_2$, and $C_3\, D_3$ must all pass through the pitch-point p.

Assuming that the arcs of approach and recess are equal, C_2 and D_2 will be the centres of the circular tooth outlines in contact at p. Mark off along $C_1\, D_1$ and $C_3\, D_3$ respectively, $C_1\, F_1$ and $C_3\, F_3$ each equal to $C_2\, p$. Then F_1 and F_3 will be the extreme points on the path of contact ; the addendum-circle of wheel B will pass through F_1, and the addendum-circle of wheel A through F_3. No working portion of the teeth will lie nearer the respective wheel centres than F_1 and F_3. Fillet-circles with centres A and B may therefore be drawn through F_1 and F_3.

The circular portion of the tooth will extend between the fillet- and addendum-circles ; the fillet, between the fillet- and root-circles, is designed as with involute or cycloidal teeth.

Internal Gear.—With internal gearing the radius of the larger wheel may be considered negative, and the value of R will also

Table XVIII.—Circular Wheel-Teeth. External Gear. $m = 0·30$.

R		1·0			1·5			2·0			3·0				4·0			5·0			10·0			
$\frac{h}{r_1}$		0·600			0·643			0·675			0·720				0·750			0·771			0·825			
n	N Approximate number of teeth in small wheel	$\frac{l_1}{r_1}$	V Per cent.	Maximum obliquity	$\frac{l_1}{r_1}$	$\frac{l_2}{r_2}$	V Per cent.	$\frac{l_1}{r_1}$	$\frac{l_2}{r_2}$	V Per cent.	$\frac{l_1}{r_1}$	$\frac{l_2}{r_2}$	V Per cent.	Maximum obliquity	$\frac{l_1}{r_1}$	$\frac{l_2}{r_2}$	V Per cent.	$\frac{l_1}{r_1}$	$\frac{l_2}{r_2}$	V Per cent.	$\frac{l_1}{r_1}$	$\frac{l_2}{r_2}$	V Per cent.	Maximum obliquity
0·30	10	1·000	3·5	17·4°	1·000	0·992	2·2	1·000	0·989	1·7	1·000	0·989	1·3	17·4°	1·000	0·989	1·1	1·000	0·990	0·97	1·000	0·994	0·76	17·4°
0·25	13	0·986	2·0	17·4	0·989	0·984	1·3	0·991	0·984	1·0	0·992	0·985	0·74	16·7	0·993	0·987	0·62	0·994	0·988	0·56	0·994	0·993	0·44	16·5
0·20	16	0·975	1·0	17·4	0·980	0·977	0·66	0·983	0·979	0·51	0·986	0·982	0·38	16·0	0·987	0·985	0·32	0·988	0·987	0·29	0·990	0·992	0·23	15·6
0·15	20	0·966	0·42	17·4	0·974	0·971	0·27	0·977	0·975	0·21	0·981	0·980	0·16	15·4	0·983	0·983	0·13	0·984	0·985	0·12	0·987	0·991	0·09	14·7
0·10	31	0·960	0·13	17·4	0·969	0·968	0·08	0·973	0·972	0·06	0·978	0·978	0·05	14·8	0·980	0·982	0·04	0·981	0·984	0·04	0·984	0·991	0·03	13·8
0·05	63	0·956	0·01	17·4	0·965	0·965	0·01	0·971	0·971	0·01	0·976	0·977	0·00	14·1	0·978	0·981	0·00	0·980	0·984	0·00	0·982	0·990	0·00	12·9
0·00	∞	0·954	0·00	17·4	0·964	0·965	0·00	0·970	0·970	0·00	0·975	0·977	0·00	13·4	0·977	0·981	0·00	0·977	0·984	0·00	0·982	0·990	0·00	12·0

Table XIX.—Circular Wheel-Teeth. Internal Gear. $m = 0·20$.

R		10·0			5·0			4·0			3·0			2·0	Maximum obliquity, for all values of R
$\frac{h}{r_1}$		0·675			0·800			0·900			1·200			—	
n	N Approximate number of teeth in small wheel	$\frac{l_1}{r_1}$	$\frac{l_2}{r_2}$	V	$\frac{l_1}{r_1}$	$\frac{l_2}{r_2}$	V	$\frac{l_1}{r_1}$	$\frac{l_2}{r_2}$	V	$\frac{l_1}{r_1}$	$\frac{l_2}{r_2}$	V	—	
0·30	10	1·007	1·004	0·62	1·005	1·016	0·42	1·004	1·020	0·32	1·003	1·081	0·19	$\frac{l_1}{r_1} = 1·000,\ \frac{l_2}{r_2} = \infty$; $V = 0·000$	14·0°
0·25	13	1·003	1·005	0·36	1·002	1·018	0·24	1·002	1·031	0·19	1·001	1·083	0·11		12·5
0·20	16	1·000	1·006	0·18	1·000	1·019	0·12	1·000	1·033	0·10	1·000	1·085	0·06		11·5
0·15	20	0·998	1·006	0·08	0·998	1·020	0·05	0·999	1·034	0·04	0·999	1·087	0·02		10
0·10	31	0·996	1·007	0·02	0·997	1·021	0·01	0·998	1·035	0·01	0·998	1·088	0·01		9
0·05	63	0·995	1·007	0·00	0·995	1·021	0·00	0·997	1·035	0·00	0·998	1·089	0·00		8
0·00	∞	0·995	1·007	0·00	0·996	1·022	0·00	0·997	1·035	0·00	0·998	1·089	0·00		7

be negative. In (12), (13), (14), and (15) substitute $R = -R$; they become respectively,

$$\frac{h}{r_1} = \frac{3(R-1)m}{R-2} \quad \ldots \ldots \ldots \quad (16)$$

$$\left(\frac{l_1}{r_1}\right)^2 = 1 - \frac{(R-2)}{3R}(m^2 - n^2) \quad \ldots \ldots \quad (17)$$

$$\left(\frac{l_2}{r_2}\right)^2 = 1 + \frac{(2R-1)^3 m^3}{3R^2(R-2)^2} - \frac{(2R-1)}{3R^2}n^2 \quad \ldots \quad (18)$$

$$V = \frac{6{\cdot}415(R-1)(R-2)}{R^2}\frac{n^3}{m} \quad \ldots \ldots \quad (19)$$

Table XIX., with $m = {\cdot}2$, is calculated from these equations.

The values of $\frac{l_1}{r_1}$, $\frac{l_2}{r_2}$, in Tables XVIII. and XIX. change so slowly, that their values corresponding to any value of R and n not found in the tables can easily be found by interpolation.

314. **Strength of Wheel-Teeth.**—The mutual pressure F between a pair of wheels is sometimes distributed over two or more teeth of each wheel; but when one of the pair has a small number of teeth it is impossible to have an arc of contact equal to twice the pitch, and the whole pressure will be borne at times by a single tooth; each tooth must therefore be designed as a cantilever fixed to the rim of the wheel and supporting a transverse load F at its point. Let p be the pitch of the teeth, b the width, h the thickness of a tooth at the root, and l the perpendicular distance from the middle of the root of the tooth to the line of action of F. Then the section at the root is subjected to a bending-moment $F\,l$, while the moment of resistance of the section is $\frac{b\,h^2 f}{6}$. Therefore,

$$F\,l = \frac{b\,h^2 f}{6} \quad \ldots \ldots \ldots \quad (20)$$

The width of the teeth is usually made some multiple of the pitch; let $b = k\,p$. The height of the tooth may also be expressed as a multiple of p; it is often as much as $\cdot 7\,p$, but since long teeth are necessarily weak, the teeth should be made as short as possible consistent with the arc of contact being at least equal to the pitch.

If the height be equal to $\cdot 6\,p$, the length l may be assumed equal to $\cdot 5\,p$. If there be no side-clearance the thickness at the pitch-line will be $\cdot 5\,p$, and with a strong tooth form the thickness at the root will be greater. Even with side-clearance, we may assume $h = \cdot 5\,p$. Substituting in (20) we have

$$F = \cdot 08333\,p^2\,k f \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

or, writing $p = \frac{\pi}{P}$, P being the diametral pitch-number,

$$F = \cdot 82246\,\frac{k f}{P^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

The value that can be taken for f, the safe working stress of the material, depends in a great measure on the conditions to which the wheels are subjected. If the teeth be accurately cut and run smoothly, they will be subjected to comparatively little shock. For steel wheels with machine-cut teeth, 20,000 lbs. per sq. in. seems a fairly low value for f the safe working stress.

Table XX. is calculated on the assumptions made above.

TABLE XX.—SAFE WORKING PRESSURE ON TOOTHED WHEELS.

Calculated from equation (22).

Pitch-number	Lbs. Pressure when $k = \frac{b}{p} =$				
	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
5	548	822	1097	1371	1645
6	381	571	761	952	1142
7	282	421	561	703	845
8	215	323	430	538	645
9	170	266	341	427	511
10	137	206	274	343	411
11	114	171	228	275	342
12	96	144	192	239	287
13	81	122	162	203	244
14	70	106	141	176	211
15	61	91	122	152	183
16	54	80	107	134	161
18	43	64	85	107	128
20	34	51	69	86	103
22	28	42	55	71	85
24	24	36	48	60	72

The arc of contact is sometimes made equal to two or three times the pitch, with the idea of distributing the total pressure over two or three teeth. But in this case, although the pressure on each tooth may be less than the total, they must be made longer in order to obtain the necessary arc of contact. It is therefore possible that when the pressure is distributed, the teeth may be actually weaker than if made shorter and the pressure concentrated on one.

In cycloidal teeth, for a given thickness at the pitch-line, the thickness at the root is greater the smaller the rolling-circle; where strength is of primary importance, therefore, a small rolling-circle should be adopted. In involute teeth, the angle of obliquity influences the thickness at the root in the same manner; the greater the angle of obliquity the greater the root thickness. In circular teeth, the greater m be taken, the thicker will be the teeth at the root.

315. **Choice of Tooth Form.**—It has already been remarked that involute toothed-wheels possess the valuable property that their centres may be slightly displaced without injury to the motion. Involute tooth outlines are simpler than cycloidal outlines, the latter having a point of inflection at the pitch-circle; involute teeth cutters are therefore much easier to make to the required shape than cycloidal. With involute teeth the direction of the line of action is always the same, but with cycloidal teeth it continually changes, and therefore the pressure of the wheel on its bearing is continually changing. Taking everything into consideration, involute teeth seem to be preferable to cycloidal. The old millwrights and engineers invariably used cycloidal, but the opinion of engineers is slowly but surely coming round to the side of involute teeth.

316. **Front-driving Gears.**—Toothed-wheel gearing has been more extensively used for front-driving bicycles than for rear-drivers. A few special forms may be briefly noticed.

'Sun-and-Planet' Gear.—In the 'Sun-and-Planet' Safety (fig. 479), the pedal-pins are not fixed direct to the ends of the main cranks, but to the ends of secondary links, hung from the crank-pin. A small pinion is fastened to each pedal-link and gears with a toothed-wheel fixed to the hub. This is a simple

form of epicyclic train, and can be treated as in section 306. If N_1 and N_2 be the numbers of teeth on the hub and pedal-link respectively, it can be shown (sec. 306) that the speed-ratio of the driving-wheel and main crank is $\frac{N_1 + N_2}{N_1}$.

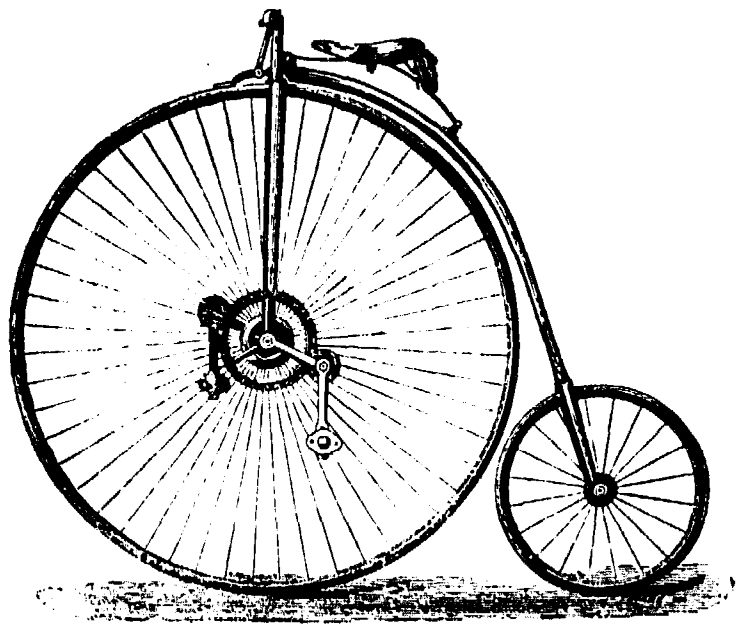


FIG. 479.

If the driving-wheel be 40 in. diameter, and the pinion and hub have 10 and 30 teeth respectively, the bicycle is geared to $\frac{30 \times 10}{30} \times 40$ in. $= 53 \cdot 3$ inches.

It should be noticed that the pedal-link will not hang vertically, owing to the pressure on the pinion. During the down-stroke the pedal will be behind the crank-pin; while on the up-stroke, if pressure be applied to the pedal, it will be in front of the crank-pin. The pedal path is therefore an oval curve with its longer axis vertical. If the pressure on the pedal be always applied vertically, the pedal path will be an ellipse, with its minor axis equal to the diameter of the toothed-wheel on the driving-hub.

This simple gear might repay a little consideration on the part of those who prefer an up-and-down to a circular motion for the pedals.

The 'Geared Facile' is a combination of the 'Facile' and 'Sun-and-Planet' gears, the lower end of the pinion-link of the latter being jointed to the pedal-lever of the former. In figure 124, the planet-pinion is 2 in. diameter, the hub-wheel 4 in. diameter, and the driving-wheel 40 in. diameter; the bicycle is therefore geared to $\frac{(4 + 2)}{4} \times 40 = 60$ in.

Perry's Front-driving Gear is similar in arrangement to the back gear of a lathe. The crank-axle (fig. 480) passes through the hub and is carried by it on ball-bearings. A toothed-wheel

fixed to the crank-axle gears with a wheel on a short intermediate spindle, to which is also fastened a wheel gearing in turn with one fastened to the hub of the driving-wheel ; the whole arrangement being the same as diagrammatically shown in figure 470.

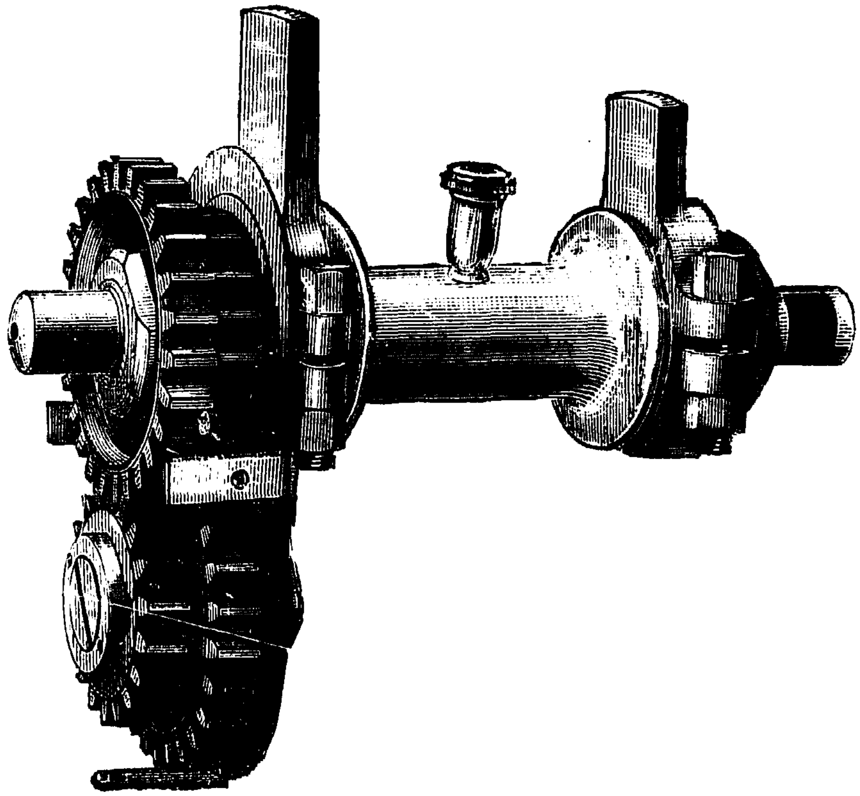


FIG. 480.

The mutual pressure between the wheels D and E (fig. 470) is equal to the tangential effort on the pedal multiplied by the ratio of the crank length to the radius of wheel D.

Example. If the pedal pressure be 150 lbs., the crank length $6\frac{1}{2}$ in., and the radius of wheel D $1\frac{1}{4}$ in., the pressure on the teeth will be $\frac{6\frac{1}{2}}{1\frac{1}{4}} \times 150 = 780$ lbs.

The 'Centric' Front-driving Gear affords an ingenious example of the application of internal contact. A large annular wheel is fixed to the crank-axle and drives a pinion fixed to the hub of the driving-wheel, the arrangement being diagrammatically shown in figure 481 ; a and b being the centres of the crank-axle and the driving-wheel hub respectively. As the crank-axle has to pass right through the hub, the latter must be large enough to encircle the former, as shown in section (fig. 482). The hub ball-races are of correspondingly large diameter, the inner race being a disc set

eccentrically to the crank-axle centre. The central part of the hub must be large enough to enclose the toothed-wheel on the

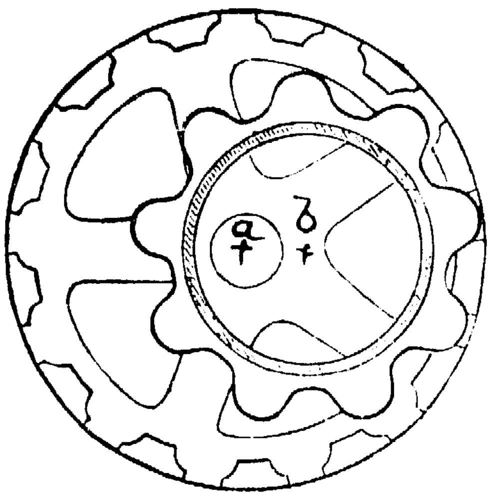


Fig. 481.

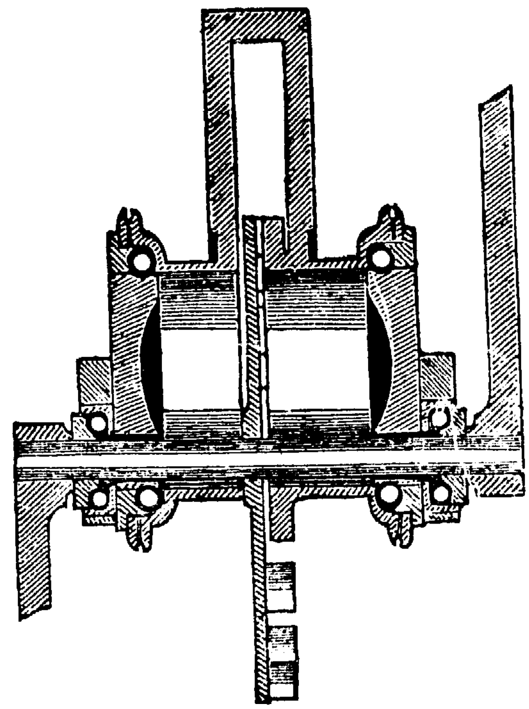


Fig. 482.

crank-axle. Instead of being made continuous and enclosing the toothed-wheel completely, the hub is divided in the middle, and the end portions are united by a triangular frame.

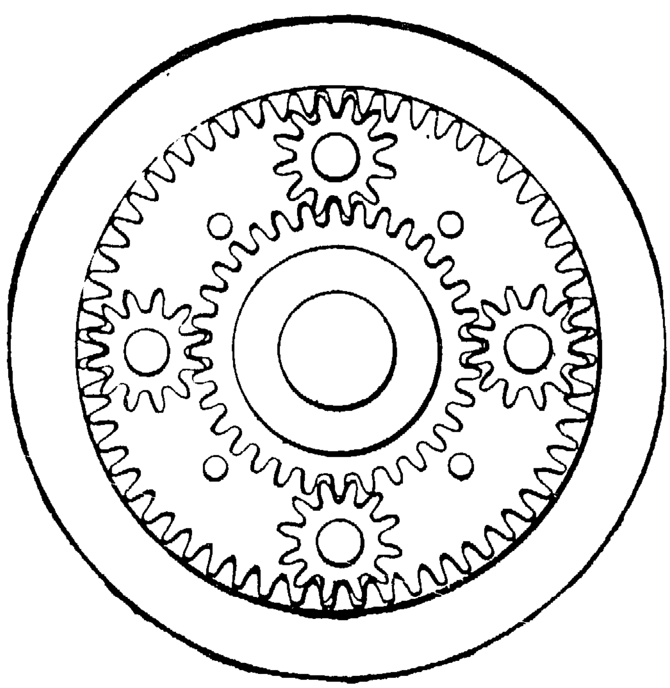


Fig. 483.

From figure 482 it is evident that the 'Centric' gear can only be used for speed-ratios of hub and crank-axle less than 2.

The 'Crypto' Front-driving Gear is an epicyclic train, similar in principle to that shown in figure 472. Figure 484 is a longitudinal section of the gear; figure 483 an end view, showing the toothed-wheels; and figure 485 an outside view of the

hub, bearings and cranks complete. The arm C (fig. 472) in this case takes the form of a disc fastened to the crank-axle A, and carrying four wheels E, which engage with the annular wheel G, forming part of the hub of the driving-wheel, and with the small wheel D, rigidly fastened to the fork. The crank-axle is carried on ball-bearings attached to the fork, the hub runs on ball-bearings on

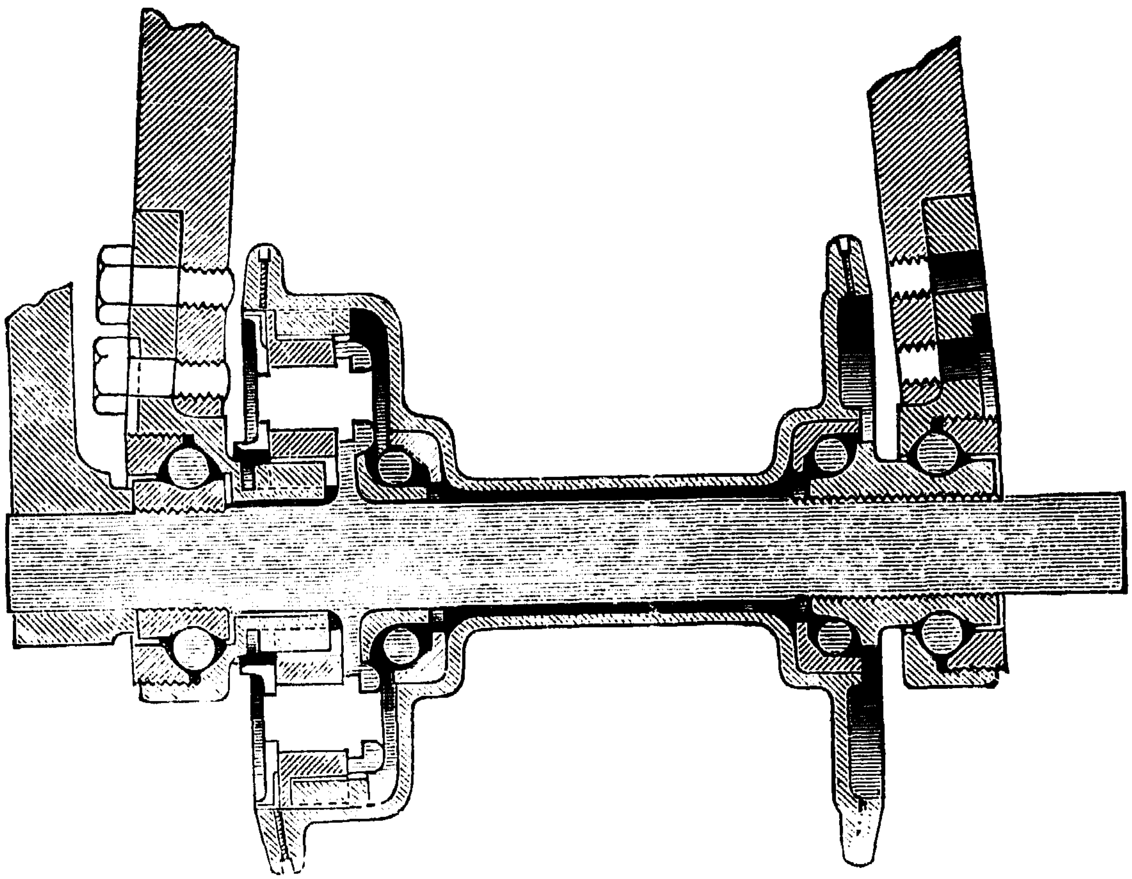


FIG. 484.

the crank-axle, while the small wheels E run on cylindrical pins B riveted to the disc C.

The pressures on the teeth of the wheels are found as follows: Considering the equilibrium of the rigid body formed by the pedal-pin, crank, crank-axle A, disc C, and pins B, the moment—about the centre of the crank-axle—of the tangential pressure P, on the pedal-pin is equal to that of the pressures of the wheels E on the pins B. Let l be the length of the crank, and r the distance of the centre of the pin B from the crank-axle; the pressure of each wheel E on its pin will be

$$\frac{1}{4} \times \frac{lP}{r} = \frac{lP}{4r} \quad . \quad . \quad . \quad . \quad . \quad (23)$$

This pressure of the pin on the wheel E is resisted by the pressures of the wheels D and G; each of these pressures must, therefore, be equal to

$$\frac{lP}{8r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

If N_1, N_2, and N_3 be the numbers of teeth on the hub-wheel G, fork-wheel D, and intermediate wheels E, respectively, the speeds of the wheels and arm C, relative to the latter, are respectively proportional to

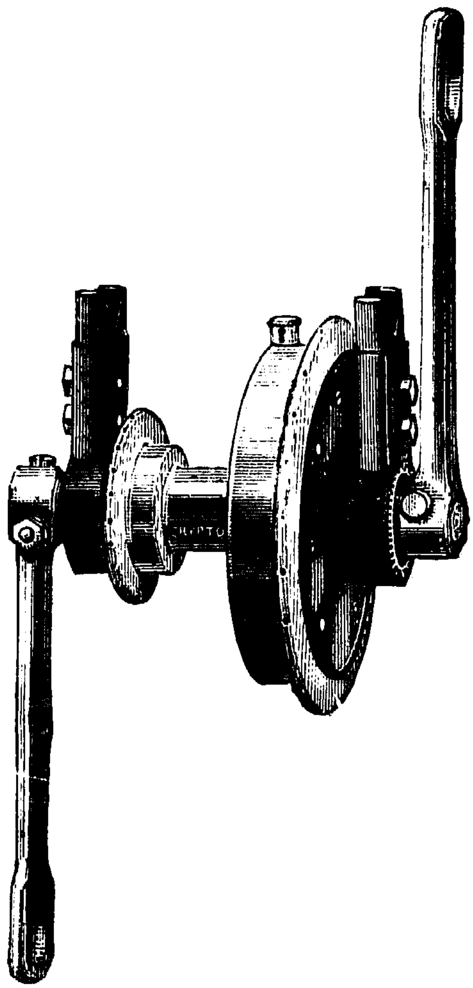


FIG. 485.

$$\frac{1}{N_1}, -\frac{1}{N_2}, \frac{1}{N_3}, \text{ and } 0 \quad . \quad . \quad (25)$$

while, relative to the fork, the speeds ω_G, ω_D, ω_E, and ω_C are proportional to

$$\left(\frac{1}{N_1}+\frac{1}{N_2}\right), 0, \left(\frac{1}{N_3}+\frac{1}{N_2}\right), \text{ and } \frac{1}{N_2} . (26)$$

Also
$$N_3 = \frac{N_1 - N_2}{2} \quad . \quad . \quad (27)$$

From (26) the speed-ratio of the hub and crank, relative to the fork, is

$$R = \frac{\left(\frac{1}{N_1}+\frac{1}{N_2}\right)}{\frac{1}{N_2}} = \frac{N_2}{N_1} + 1 \quad . \quad (28)$$

From (28) it is evident that when the hub speed is to be more than twice that of the crank, N_1 must be less than N_2; that is, the annular wheel must be fixed to the fork.

From (25) and (27) the speed-ratio of wheels E and D, relative to the disc C, is

$$-\frac{N_2}{N_3} = \frac{2N_2}{N_2 - N_1} = 2\frac{(R-1)}{(R-2)} \quad . \quad . \quad . \quad (29)$$

But the wheel D makes -1 turn relative to the crank while the latter makes 1 turn relative to the fork. Therefore, for every

turn of the crank, the wheels E make $2\dfrac{(R-1)}{(R-2)}$ turns in their bearings. Since these are plain cylindrical bearings, and the pressure on them is large, their frictional resistance will be the largest item in the total resistance of the gear.

Example.—If $l = 6\frac{1}{2}$, $r = 1\frac{1}{4}$in., and $P = 150$ lbs.; from (23) the pressure on the teeth is $\dfrac{6\frac{1}{2} \times 150}{8 \times 1\frac{1}{4}} = 97{\cdot}5$ lbs., and of the wheels E on their pins 195 lbs. Also if $R = 2{\cdot}5$, as in gearing a 28-inch driving-wheel to 70-inch, the wheels E each make $\dfrac{2 \times 1{\cdot}5}{{\cdot}5} = 6$ turns on their pins to one turn of the crank.

317. **Toothed-wheel Rear-driving Gears.**—A number of gears have been designed from time to time with the object of replacing the chain, but none of them have attained any considerable degree of success.

The 'Burton' Gear was a spur-wheel train, consisting of a spur-wheel on the crank-axle, a small pinion on the hub, and an intermediate wheel, gearing with both the former and running on an intermediate spindle on the lower fork. The intermediate wheel did not in any way modify the speed-ratio, so that the gearing up of the cycle depended only on the numbers of teeth of the wheels on the crank-axle and hub respectively. If r was the radius of the spur-wheel on the crank-axle and l the length of the crank, the upward pressure on the teeth of the intermediate wheel was $\dfrac{lP}{r}$, and therefore the upward pressure of the intermediate wheel on its spindle was $2\dfrac{lP}{r}$. This upward pressure was so great, that an extra bracing member was required to resist it.

Example.—If $P = 150$ lbs., $l = 6\frac{1}{2}$ in., $r = 4$, the pressure on the intermediate spindle $= \dfrac{2 \times 6\frac{1}{2} \times 150}{4} = 487{\cdot}5$ lbs.

The Fearnhead Gear was a bevel-wheel gear, bevel-wheels being fixed on the crank-axle and hub respectively and geared together by a shaft enclosed in the lower frame tube. If bevel-wheels could be accurately and cheaply cut by machinery, it is possible that

gears of this description might supplant, to a considerable extent, the chain-driving gear ; but the fact that the teeth of bevel-wheels cannot be accurately milled is a serious obstacle to their practical success.

318. **Compound Driving Gears.**—For front-driving, Messrs. Marriott and Cooper used an epicyclic train (fig. 486), formed from a pair of spur-wheels and a pair of chain-wheels. Two spur-wheels, *D* and *E*, rotate on spindles fixed to the crank *C*. Rigidly fixed to *E* is a chain-wheel *G*, connected by a chain to a chain-wheel *F*, fixed to the fork. If the arm *C* be fixed and the pinion *D* be rotated, the chain-wheel *F* will be driven in the opposite direction. Let $-n$ be the speed-ratio of the wheels *F* and *D* relative to the arm *C* (in figure 486, $-n = -1$), then the angular speeds of *F*, *C*, and *D* are respectively proportional to n, 0, and -1. If a rotation $+1$ be given to the whole system, their speeds will be proportional to $(n+1)$, 1 and 0 respectively. The wheel *D* is fixed to the fork, the wheel *F* to the hub of the driving-wheel, and *C* is the crank. The driving-wheel, therefore, makes $(n+1)$ turns to one turn of the crank. With this gear, any speed-ratio of driving-wheel and crank can be conveniently obtained.

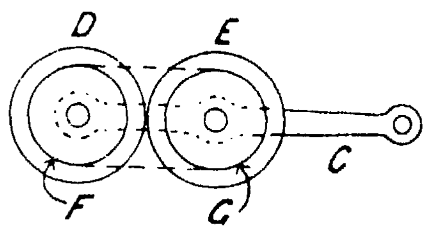


FIG. 486.

A number of compound rear-driving gears have been made, some of which have been designed with the object of avoiding the use of a chain. In 'Hart's' gear, a toothed-wheel was fixed on the crank-axle and drove through an intermediate wheel a small pinion ; a crank fixed on this pinion was connected by a coupling-rod to a similar crank on the back hub. In this gear, there was a dead-centre when the hub crank was horizontal, and when going up-hill at a slow pace the machine might stop. In 'Devoll's' gear the secondary axle was carried through to the other side of the driving-wheel, two coupling-rods and pairs of cranks were used, and the dead-centre avoided.

The 'Boudard' Gear (fig. 487) was the first of a number of compound driving gears in which the chain is retained. An annular wheel is fixed near one end of the crank-axle and gears with a pinion on a secondary axle ; at the other end of the

secondary axle a chain-wheel is fixed and is connected by a chain in the usual way to a chain-wheel on the back hub. A great deal of discussion has taken place on the merits and demerits of this gear; probably its promoters at first made extravagant claims, and its opponents have overlooked some points that may be advanced in its favour. Of course, the mere introduction of an additional axle and a pair of spur-wheels is rather a disadvantage on account of the extra friction. In the chapter on Chain Gearing it has been shown that it is advantageous to make the chain run at a high speed; this can be done with the ordinary chain gearing by making both chain-wheels with large numbers of teeth, but if the back hub chain-wheel be large, say with twelve teeth, that on the crank-axle must be so large as to interfere with the arrangement of the lower fork. The 'Boudard' gear is a convenient means of using a high gear with a large chain-wheel on the back hub.

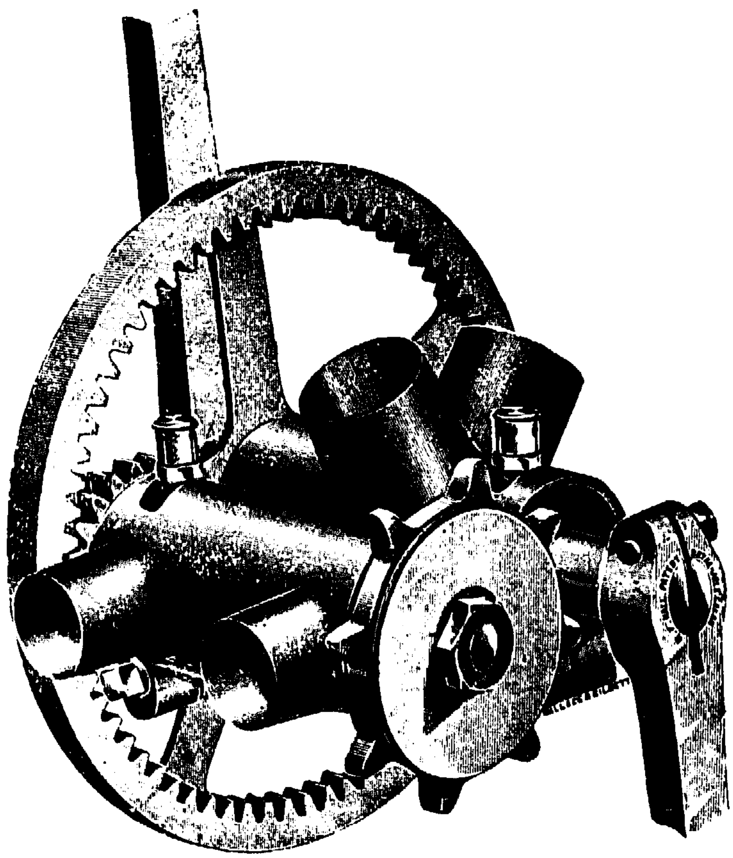


FIG. 487.

The 'Healy' Gear (fig. 488) is an epicyclic bevel gear having a speed-ratio of 2 to 1, which has the advantage of being more compact than the 'Boudard' gear, but has the disadvantage which applies to all bevel-wheels, viz. the fact that they cannot be cheaply and accurately cut.

Geared Hubs.—Compound chain gears have been used in which the toothed-wheel gearing is placed at the hub of the driving-wheel. In the 'Platnauer' gear (fig. 489) the small pinion is fixed to the hub and gears with a large annular wheel which runs on a disc set eccentrically to the hub spindle, a row of balls being

introduced. The outer part of this wheel has projecting teeth to gear with the chain.

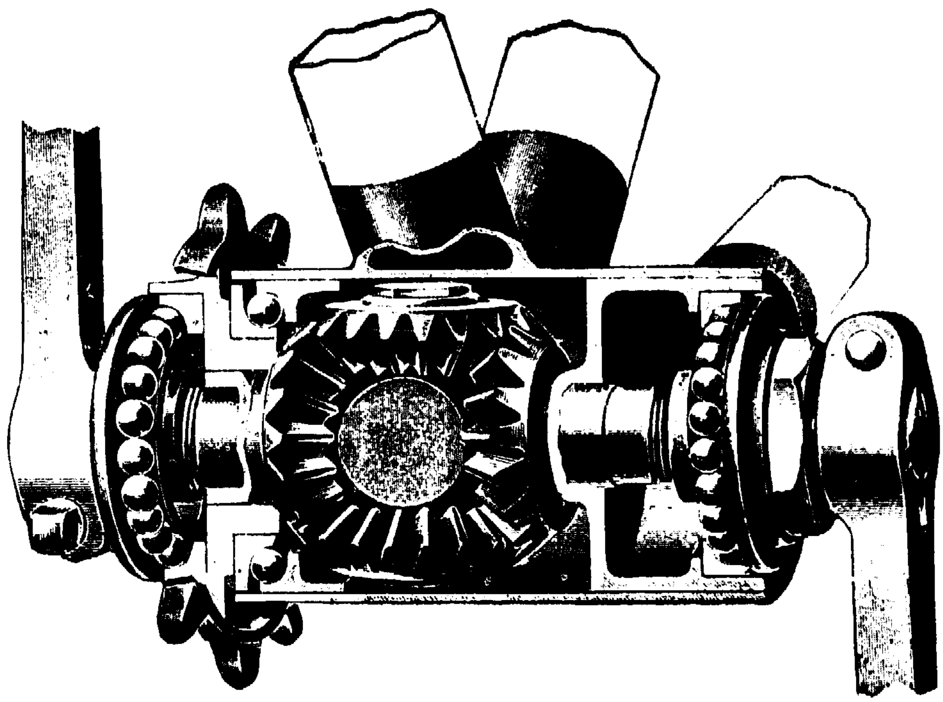


FIG. 488.

These hub gears, as far as we can see, have none of the advantages of the crank-axle gears to recommend them, since the speed of the chain cannot be increased unless a very large crank-

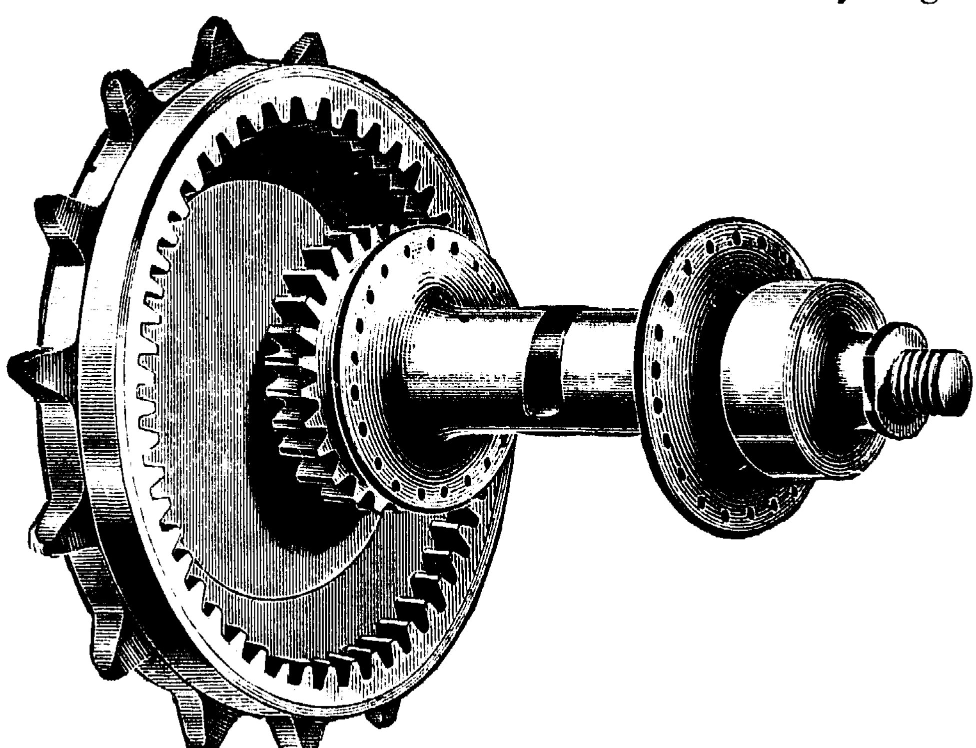


FIG. 489.

axle chain-wheel be used, and they possess the disadvantages of additional frictional resistance of the extra gear

319. **Variable Speed Gears.**—It has been shown, in Chapter XXI., that it is theoretically desirable to lower the gear of the cycle while riding up-hill.

In the '*Collier*' *Two-Speed Gear*, of which figure 490 is a section, and figure 491 a general sectional view, a stud-wheel *D* (that is, a wheel with pin teeth) fixed on the crank-axle gears with a toothed-pinion *P* attached to the chain-wheel *C*. The crank-axle *A* is carried on a hollow axle *B*, the axes of the two axles

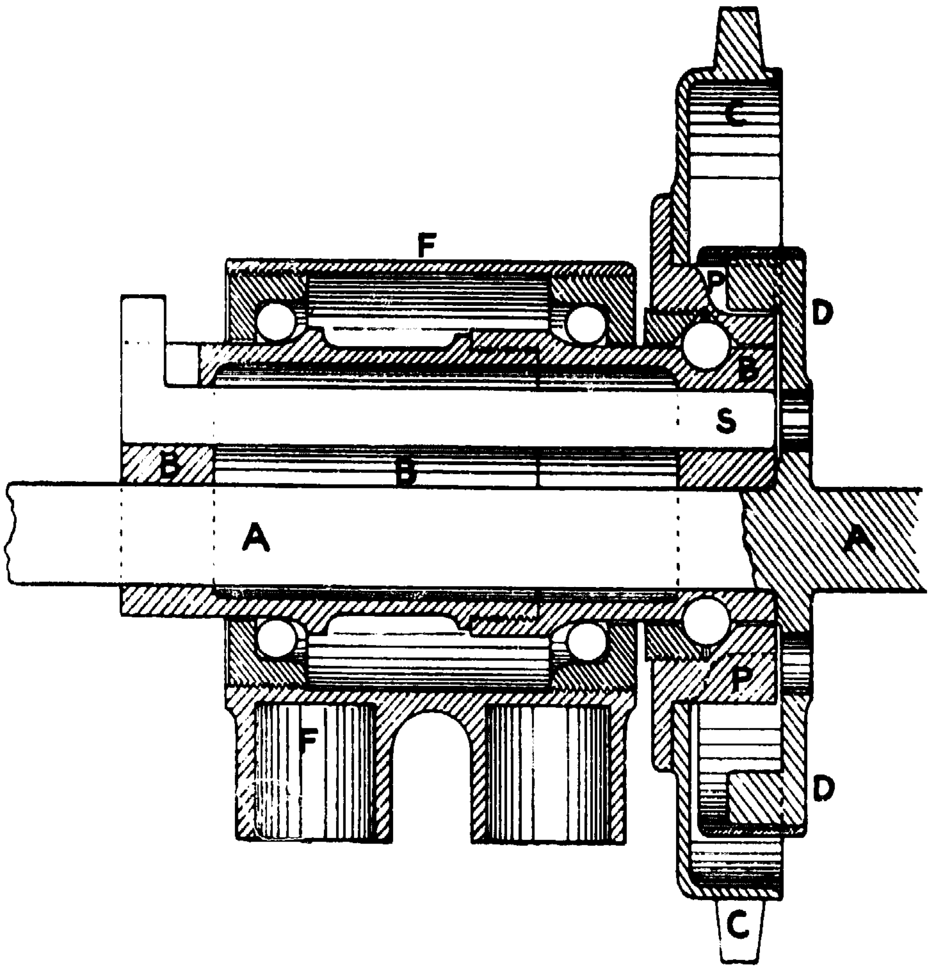


FIG. 490.

being placed eccentrically. The chain-wheel *C*, and with it the toothed-pinion *P*, revolves on a ball-bearing at the end of the hollow axle *B*. There are twelve and fifteen teeth respectively on the pinion and stud-wheel, so that the ratio of the high and low gears is 5 : 4. When the low gear is in use, the two axles are locked together by means of a slide bolt *S* in the hollow axle which engages with a hole in the stud-wheel *D*, the whole revolving together on ball-bearings in the bottom-bracket *F*. When the high gear is used, the bolt in the hollow axle is with-

drawn from the hole in the stud-wheel and fits in a notch in the operating lever. The toothed-pinion, and with it the chain-wheel C, is then driven at a higher speed than the crank-axle.

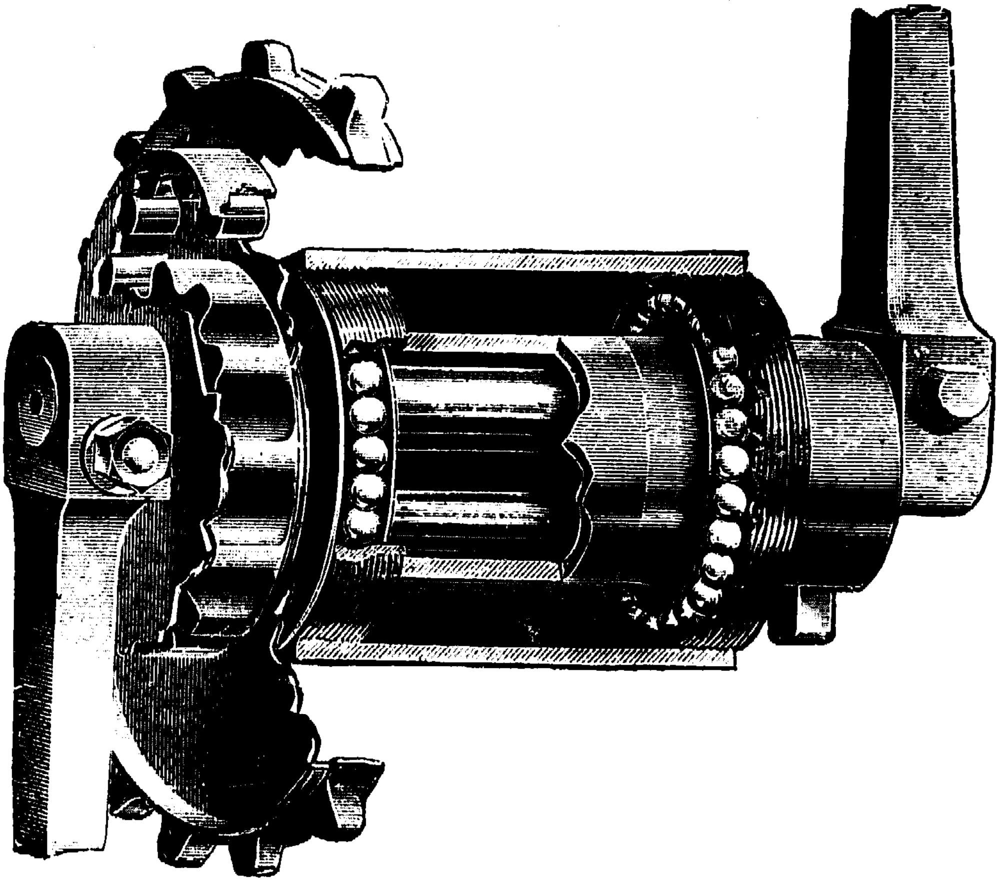


FIG. 491.

The arrangement of the two axles is shown diagrammatically in figure 492. When the high gear is in use the centre b of the crank-axle is locked in position vertically about the centre a of the hollow axle. If the cranks are exactly in line at high gear, the virtual cranks $a\ c_1$ and $a\ c_2$ will be slightly out of line at low gear. The pedals, however, describe practically equal circles with either gear in use.

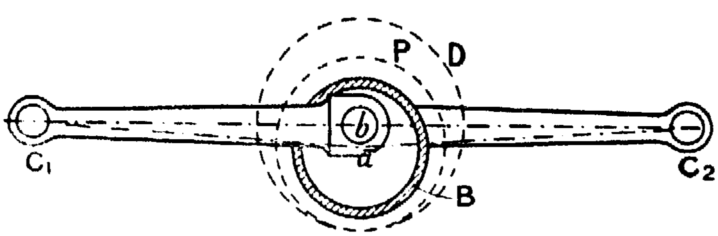


FIG. 492.

The 'Eite and Todd' Two-Speed Gear (fig. 493) consists of a double-barrelled bracket carrying the crank-axle—on which is keyed a toothed-wheel—and a secondary axle, to which is fixed two small pinions at one end, and the chain-wheel at the other. The pinions on the secondary axle are in gear with intermediate

pinions running on balls on adjustable studs attached to an arm which can swing round the secondary axle. One or other of the intermediate pinions can be thrown into gear with the spur-wheel on the crank-axle, by the shifting mechanism under the control of the rider, by means of a lever placed close to the handle-bar.

The Cycle Gear Company's Two-Speed Gear has an epicyclic train somewhat similar in principle to that of the 'Crypto' front-

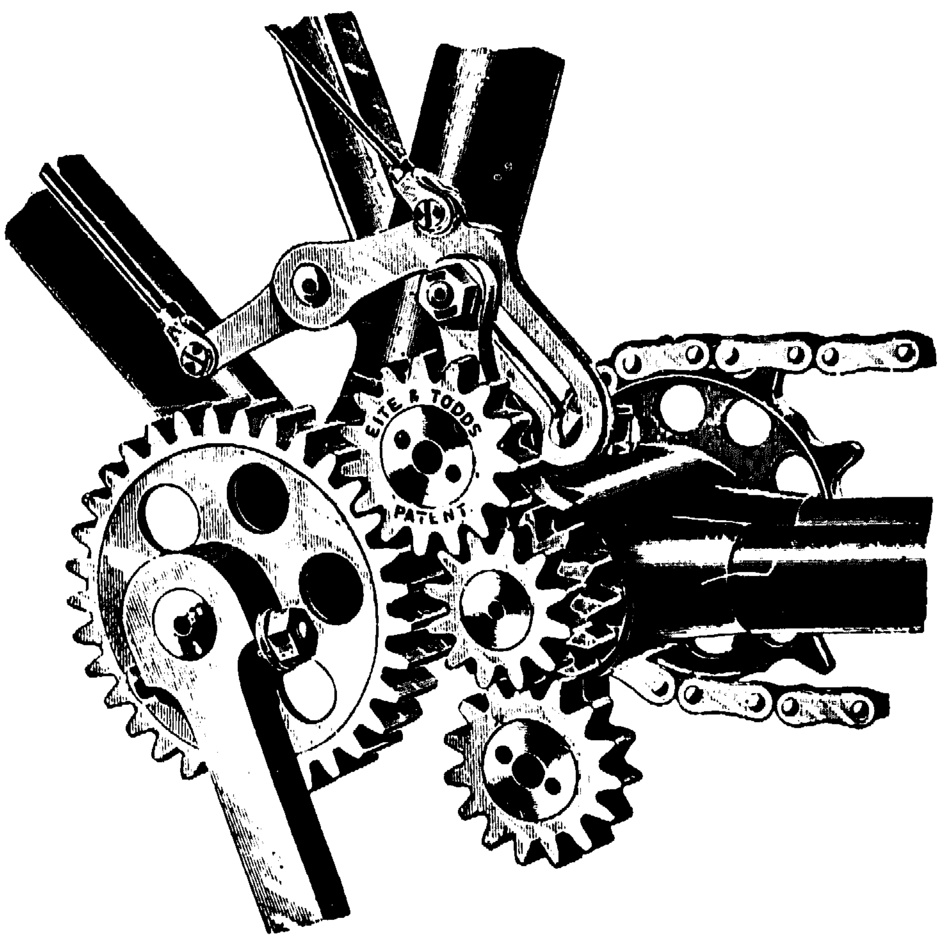


FIG. 493.

driving gear. When high speed is required, the whole of the gear rotates as one rigid body; but when low speed is required the small central wheel is fixed and the chain-wheel driven by an epicyclic train.

The same Company also make a two-speed gear, the change of gearing being effected at the hub of the driving-wheel.

The 'J. and R.' Two-Speed Gear (fig. 494) consists of an epicyclic gear in the back hub; the central pinion of the gear is fixed to the driving-wheel spindle when the low gear is used, the wheel hub then rotating at a slower speed than the

chain-wheel. When the high gear is used, the epicyclic gear—and with it, of course, the chain-wheel—is locked to the driving-wheel hub. C_1 is the main portion of the driving-wheel hub; to this is fastened the end portion C_2, on which are formed a ball-race for the chain-wheel G, and an annular wheel D_2 in which the

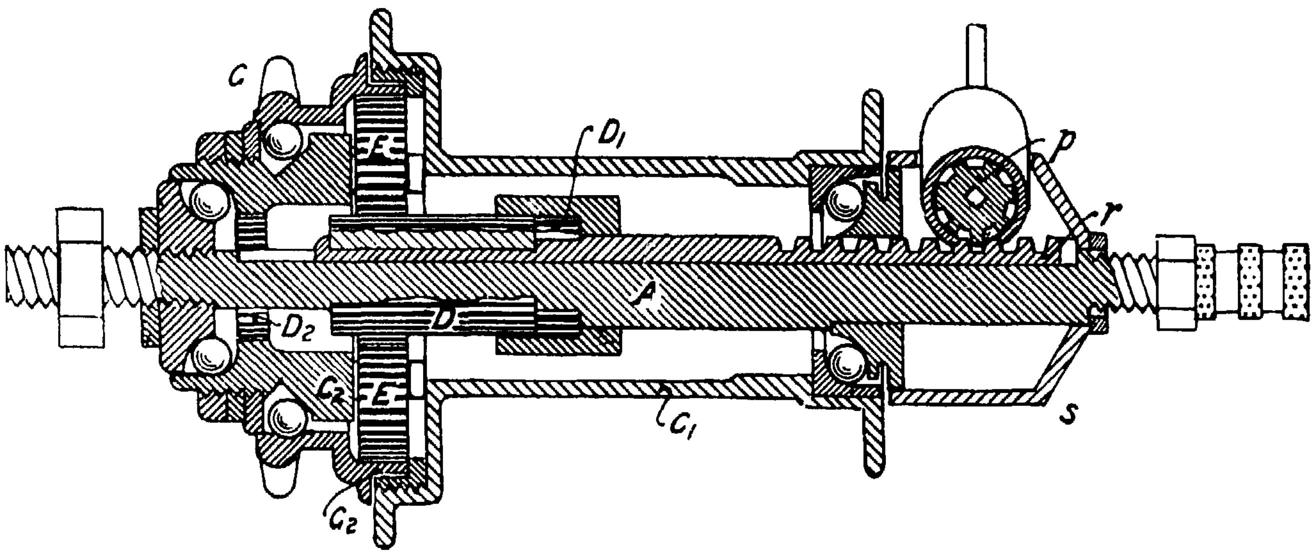


FIG. 494.

central pinion D can be locked. The intermediate pinions E, four in number, revolve on pins fastened to the hub C_1 and C_2. The annular wheel G_2, which gears with the intermediate pinions, is made in one piece with the chain-wheel G. When the low gear is in use the central pinion D is held by the axle-clutch D_1

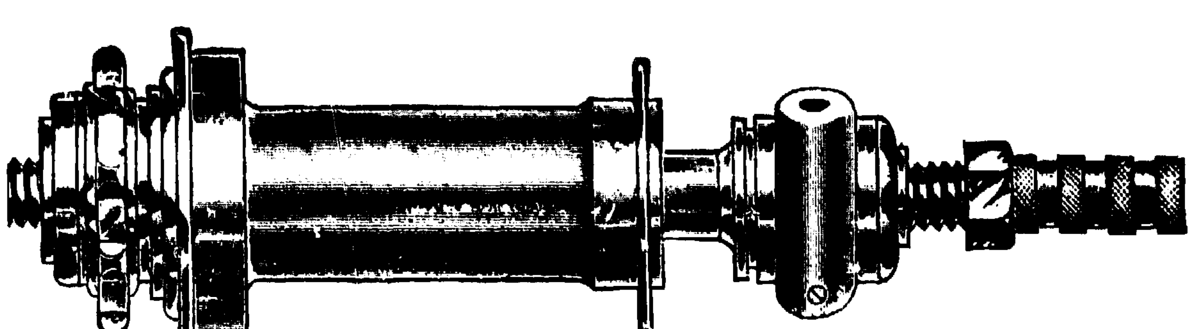


FIG. 495.

fastened to the spindle A. To change the gear, the central pinion D is shifted longitudinally out of gear with the axle clutch and into gear with the annular wheel D_2. This shifting is done by means of a rack r and pinion p; the latter is supported in a shifter-case S fixed to the driving-wheel spindle, and is operated by the rider at pleasure.

Figure 495 shows an outside view of the hub with the spindle and shifter-case partially removed.

The 'Sharp' Two-Speed Gear (fig. 496) is an adaptation of the 'Boudard' driving gear. On the crank-axle A the disc D_1 carries a drum D_2 on which are formed two annular wheels w_1 and w_2 which can gear with pinions p_1 and p_2 fastened to the secondary axle. The secondary axle is in two parts ; the chain-wheel W is

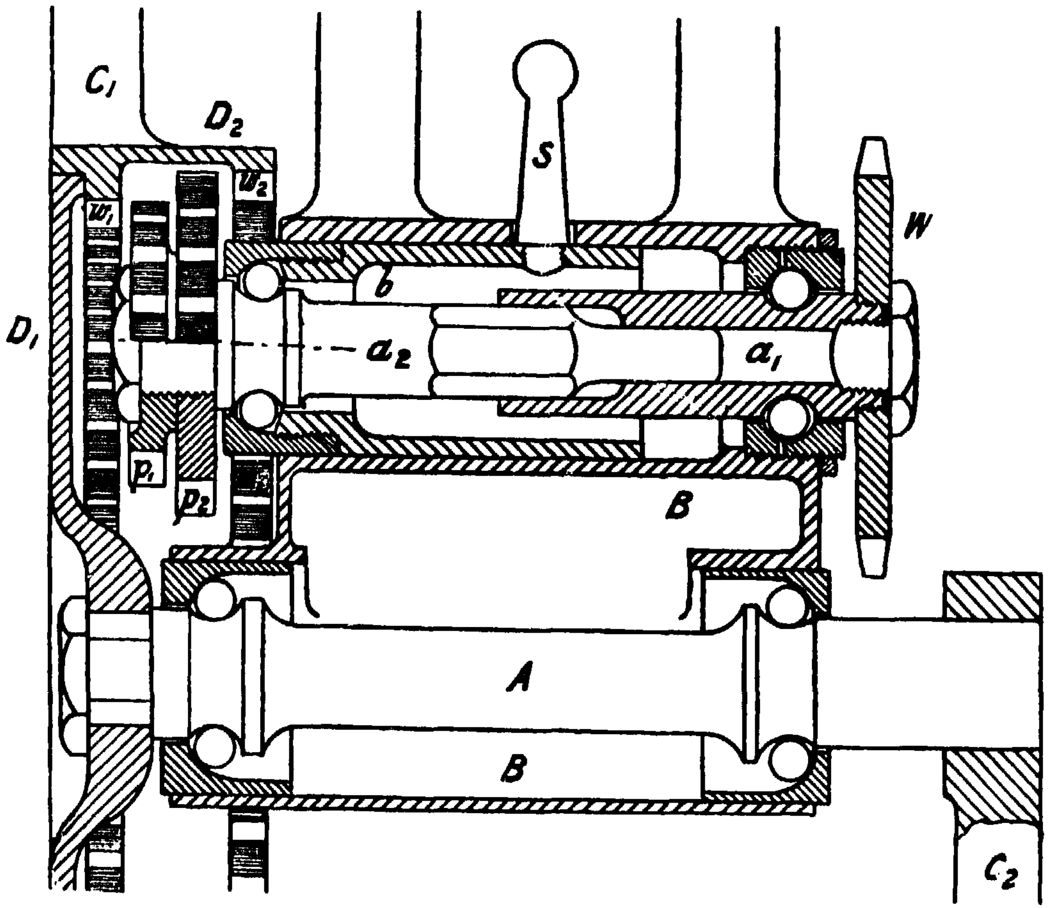


FIG. 496.

fixed to one part a_1, the pinions p_1 and p_2 to the other part a_2. The ball-bearing near the end of a_2 is carried by a secondary bracket b, which can be moved longitudinally in the main bracket B, so that the pinion p_1 may be moved into gear with the wheel w_1, or pinion p_2 into gear with wheel w_2 ; while in the intermediate position, shown in the figure, the crank-axle may remain stationary while the machine runs down hill. A hexagonal surface on the portion a_2 fits easily in a hollow hexagonal surface on the portion a_1 of the secondary axle, so that the one cannot rotate without the other, although there is freedom of longitudinal movement. The longitudinal movement is provided by a stud s,

which passes through a small spiral slot in the main bracket B, and is screwed to the inner movable bracket b. The end of the stud can be raised or lowered, and the sliding bracket simultaneously moved longitudinally, by the rider, by means of suitable mechanism, and the gear changed from high to low, or *vice versa*.

The drum D_2 is wider than that on the ordinary 'Boudard' gear, the corresponding crank, C_1, may therefore be fastened to the outside of the drum instead of to the end of the crank-axle. The other crank, C_2, is fastened in the usual way to the crank-axle.

Linley and Biggs' Expanding Chain-wheel (fig. 465) provides for three or four different gearings, and though there is no toothed-wheel gear about it, it may be mentioned here, since it has the same function as the two-speed gears above described. The rim of the chain-wheel on the crank-axle can be expanded and contracted by an ingenious series of latches and bolts, so as to contain different numbers of cogs. When pedalling ahead the driving effort is transmitted direct from the crank-axle to the chain-wheel; but if the chain-wheel be allowed to overrun the crank-axle the series of changes is effected in the former. The right pedal being above, below, before, or behind the crank-axle, corresponds to one particular size of the chain-wheel; if pedalling ahead be begun from one of these positions, the chain-wheel will remain unaltered. The length of chain is altered by the changes, therefore a loose pulley at the end of a light lever, controlled by a spring (fig. 465), is used to keep it always tight. Back-pedalling is impossible with this expanding chain-wheel, so a very powerful brake is used in conjunction with it.

A two-speed gear, with the gearing-down done at the hub, will be better than one with the gearing-down done at the crank-bracket, in so far that when driving with the low gear the speed of the chain will be greater, and therefore the pull on it will be less, presuming that the number of teeth on the back-hub chain-wheel is the same in both cases.

The frictional resistance of an epicyclic two-speed gear is probably much greater than that of an annular toothed-wheel gear, such as the 'Collier' or 'Sharp,' on account of the intermediate pinions revolving on plain cylindrical bearings under

considerable pressure. The crank-axle of the former gear runs on plain cylindrical bearings when the gear is in action. The 'Sharp' and the 'Eite and Todd' two-speed gears have the disadvantage, compared with the others, that the additional gear and its consequent increased frictional resistance is always in action; in this respect the former is exactly on a level with the ordinary 'Boudard' gear.

CHAPTER XXVIII

LEVER-AND-CRANK GEAR

320. **Introductory.**—A number of lever-and-crank gears have been used to transmit power from the pedal to the driving-axle of a bicycle; the majority of them are based on the four-link kinematic chain. In general, a lever-and-crank gear does not lend itself to gear up or down; that is, the number of revolutions made by the driving-axle is always equal to the number of complete up-and-down strokes made by the pedal. When gearing up is required, the lever-and-crank gear is combined with a suitable toothed-wheel mechanism, generally of the 'Sun-and-Planet' type. The four-link kinematic chain generally used for this gear consists of: (1) the fixed link, formed by the frame of the machine; (2) the crank, fastened to the axle of the driving-wheel, or driving the axle by means of a 'Sun-and-Planet' gear; (3) the lever, which oscillates to and fro about a fixed centre; (4) the coupling-rod, connecting the end of the crank to a point on the oscillating lever.

Lever-and-crank gears may be subdivided into two groups, according as the pedal is fixed to the lever, or to the coupling-rod of the gear. In the former group, the best known example of which is the 'Facile' gear, the pedal oscillates to and fro in a circular arc, having a dead-point at the top and bottom of the stroke. In the latter group, of which the 'Xtraordinary' and the 'Claviger' were well-known examples, the pedal path is an elongated oval curve, the pedal never being at rest relative to the frame of the machine.

With lever-and-crank gears it is easy to arrange that the down-stroke of the knee shall be either quicker or slower than the up-stroke. In the examples analysed in this chapter, where a

difference exists, the down-stroke is the quicker. Probably this is merely incidental, and has not been a result specially aimed at by the designers. Regarded merely as a mechanical question, it is immaterial whether the positive stroke be performed more quickly or slowly than the return stroke, though, possibly, physiological considerations may slightly modify the question.

321. **Speed of Knee-Joint with 'Facile' Gear.**—If the pedal be fixed to the oscillating lever, its varying speed can be found as in section 33, the speed of the crank-pin being considered constant. The speed of the knee-joint can be found as follows: Let $A\ B\ C\ D$ (fig. 497) be the four-link kinematic chain, $D\ C$ being the frame-link, $D\ A$ the crank, $C\ B$ the oscillating lever, and $A\ B$ the coupling-rod. Let the pedal be fixed to a prolongation of the oscillating lever at P. Let H and K be the rider's hip- and knee-joints respectively, corresponding to the points C and B of figure 21. In any position of the mechanism produce $D\ A$ and $B\ C$ to meet at I; I is the instantaneous centre of rotation of $A\ B$. Let $H\ K$ and $P\ C$, produced if necessary, meet at J. Since P is at the instant moving in a direction at right angles to $C\ P$, it may be considered to rotate about any point in $I\ P$; for a similar reason, K may be considered to rotate about any point in $H\ K$; therefore J is the instantaneous centre of rotation of the rider's leg, $P\ K$, from the knee downwards. Let v_a, v_b, . . . be the speeds at any instants of the points A, B, . . . Draw $D\ e$, parallel to $B\ C$, meeting $B\ A$, produced if necessary, at e.

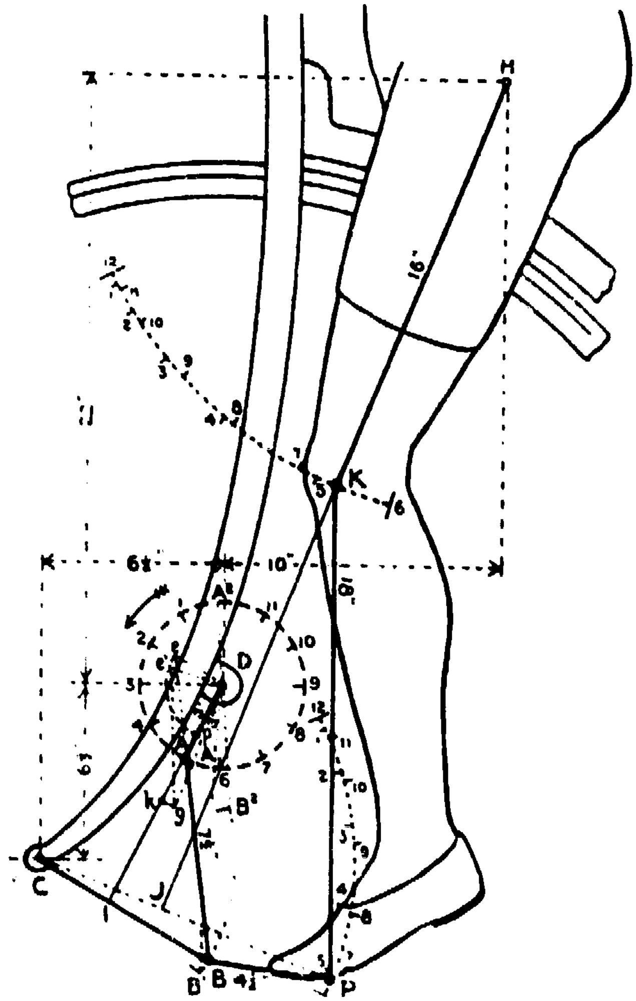


FIG. 497.

Then, since the points B and P are both rotating about the centre C,

$$\frac{v_p}{v_b} = \frac{C\,P}{C\,B} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

But from section 32,

$$\frac{v_b}{v_a} = \frac{D\,e}{D\,A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Draw $D\,e^1$ parallel to $P\,C$ and equal to $D\,e$. Draw $D\,g$ and $e^1\,g$, meeting at g, respectively parallel to $H\,K$ and $P\,K$. Since the triangles $J\,K\,P$ and $D\,g\,e^1$ are similar,

$$\frac{J\,K}{J\,P} = \frac{D\,g}{D\,e^1},$$

and

$$\frac{v_k}{v_p} = \frac{J\,K}{J\,P} = \frac{D\,g}{D\,e^1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Multiplying (1), (2), and (3) together, we get

$$\frac{v_p}{v_b} \cdot \frac{v_b}{v_a} \cdot \frac{v_k}{v_p} = \frac{C\,P}{C\,B} \cdot \frac{D\,e}{D\,A} \cdot \frac{D\,g}{D\,e^1},$$

that is, remembering that $D\,e$ and $D\,e^1$ are equal,

$$\frac{v_k}{v_a} = \frac{C\,P}{C\,B\,.\,D\,A} \cdot D\,g \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Therefore since the lengths $C\,P$, $C\,B$, and $D\,A$ are constant for all positions of the mechanisms, the speed of the knee-joint is proportional to the intercept $D\,g$. If $D\,k$ be set off along $D\,A$ equal to $D\,g$, the locus of k will be the polar speed-curve of the knee-joint.

322. **Pedal and Knee-Joint Speeds with 'Xtraordinary' Gear.**—If the pedal P be rigidly fixed to a prolongation of the coupling-rod $B\,A$, the construction is as follows: Produce $D\,A$ and $C\,B$, to intersect at I (fig. 498), the instantaneous centre of rotation of the coupling-rod $A\,B$. Draw $D\,e$, parallel to $I\,P$, cutting $A\,P$, produced if necessary, at e. [In some positions of the mechanism the instantaneous centre I will be inaccessible, and the direction of $I\,P$ not directly determinable; the following

modification in the construction may be used: Draw $D\ e^1$ parallel to $B\ C$, meeting $A\ B$ at e^1; then draw $e^1\ e$ parallel to $B\ P$, meeting $A\ P$ at e.]
Then

$$\frac{v_p}{v_a} = \frac{I\,P}{I\,A} = \frac{D\,e}{D\,A} \quad . \quad (5)$$

or,

$$v_p = \frac{v_a}{D\,A} \cdot D\,e \quad . \quad (6)$$

$D\ A$ is, of course, of constant length for all positions of the mechanism, and if the speed of the bicycle be uniform, v_a is constant, and therefore the speed of the pedal P along its path is proportional to the intercept $D e$. If $D p$ be set off along the crank $D\ A$, equal to this intercept, the locus of p will be the polar curve of the pedal's speed.

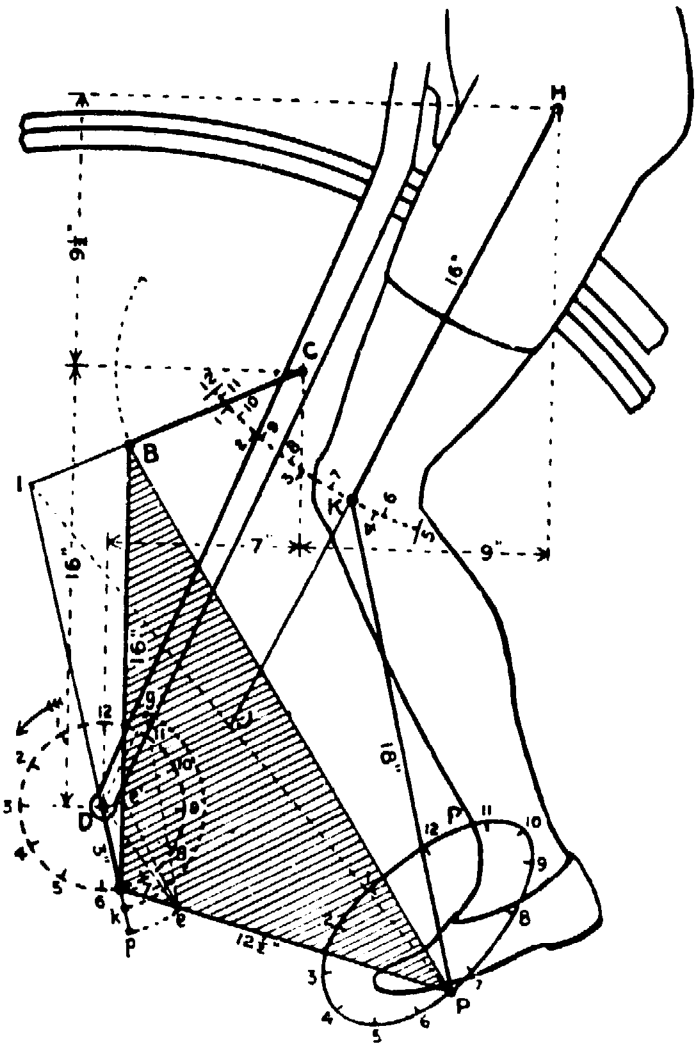


FIG. 498.

Produce $H\ K$ to meet $I\ P$ at J, then J is the instantaneous centre of rotation of the rider's leg $K\ P$ from the knee to the pedal. From D and e draw $d\ g$ and $e\ g$, meeting at g, respectively parallel to $K\ H$ and $P\ K$. Since the points K and P are at the instant rotating about the centre J,

$$\frac{v_k}{v_p} = \frac{J\,K}{J\,P} = \frac{D\,g}{D\,e} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Multiplying (5) and (7) together we get

$$\frac{v_k}{v_a} = \frac{D\,g}{D\,A}$$

or,

$$v_k = \frac{v_a}{D\,A} \cdot D\,g \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Therefore, since v_a and $D\,A$ are constant, the speed of the knee-joint is proportional to the intercept $D\,g$. If $D\,k$ be set off along the crank $D\,A$, equal to $D\,g$, the locus of k will be the polar curve of the speed of the knee-joint.

323. **Pedal and Knee-Joint Speeds with 'Geared-Facile' Mechanism.**—If toothed gearing be used in conjunction with a lever-and-crank gear, the motion of the mechanism is altered considerably. The toothed gearing usually employed in such cases is the well-known 'Sun-and-Planet' wheels, one toothed-wheel being fixed to the hub of the driving-wheel, the other centred on the crank-pin, and rigidly fixed to the coupling-rod of the gear. The driving-wheel will, as before, rotate with practically constant speed, since the whole mass of the machine and rider, moving horizontally, acts as a flywheel steadying the motion. Thus the sun-wheel of the gear moves with constant speed relative to the frame, but the speed of the crank is not constant, on account of the oscillation of the coupling-rod and planet-wheel.

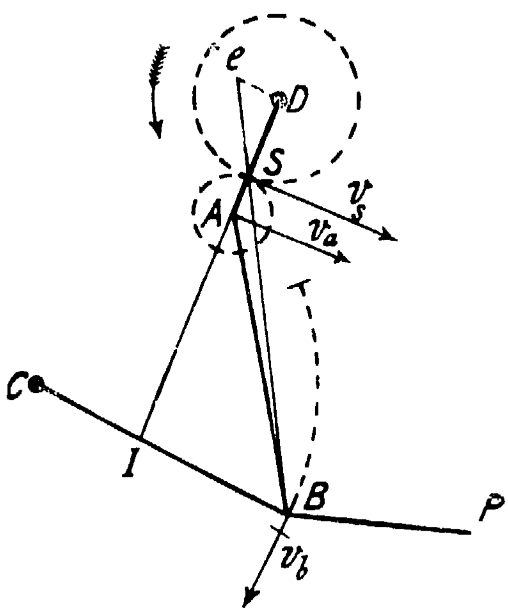


FIG. 499.

Let $D\,A\,B\,C$ (fig. 499) be, as before, the lever-and-crank gear, and let the 'Sun-and-Planet' wheels be in contact at the point S, which must, of course, lie on the crank $D\,A$. Let I be the instantaneous centre of rotation of the coupling-rod $A\,B$, and planet-wheel; and let v_s be the speed, relative to the frame, of the pitch-line of the sun-wheel; this will be, of course, the speed of the points of the wheel in contact at S. Draw $D\,e$ parallel to $C\,B$, meeting $B\,S$, produced if necessary, at e. Then,

$$\frac{v_b}{v_s} = \frac{I\,B}{I\,S} = \frac{D\,e}{D\,S}$$

or,

$$v_b = \frac{v_s}{D\,S} \cdot D\,e \qquad \qquad (9)$$

That is, the speed of the pedal is proportional to the intercept

$D\,e$, since v_s and $D\,S$ are constant. Performing the remainder of the construction as in figure 497, we get

$$\frac{v_k}{v_s} = \frac{C\,P}{C\,B\,.\,D\,S}\,.\,D\,g \qquad . \quad . \quad . \quad . \quad . \; (10)$$

The variation in the speed of the crank can easily be shown thus: The points A and S of the planet-wheel are at the instant rotating about the point I. Therefore,

$$\frac{v_a}{v_s} = \frac{I\,A}{I\,S} = 1 - \frac{A\,S}{I\,S} \qquad . \quad . \quad . \quad . \quad . \; (11)$$

$I\,S$ being considered negative when S lies between A and I.

324. **Pedal and Knee-joint Speeds with 'Geared Claviger' Mechanism.**—In this case the modification of the construction in figure 498 is the following: Let S (fig. 500) be the point of contact of the 'Sun-and-Planet' wheels. Join $P\,S$, and draw $D\,e$ parallel to $I\,P$, meeting $P\,S$ in e. Then, as in section 323,

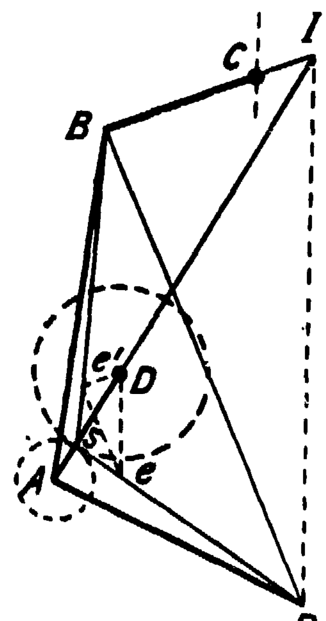


FIG. 500.

$$\frac{v_p}{v_s} = \frac{I\,P}{I\,S} = \frac{D\,e}{D\,S},$$

or,

$$v_p = \frac{v_s}{D\,S}\,.\,D\,e \quad . \quad . \quad . \quad (12)$$

That is, the pedal speed is proportional to the intercept $D\,e$.

If the instantaneous centre I of the coupling-rod be inaccessible, the method of determining $D\,e$ may be as follows:—Join $S\,B$, and draw $D\,e^1$ parallel to $C\,B$, meeting $S\,B$ at e^1. Draw $e^1\,e$ parallel to $B\,P$, meeting $S\,P$ at e.

325. **'Facile' Bicycle.**—Figure 497 represents the 'Facile' mechanism. From the centre of the driving-wheel D with radius $(D\,A + A\,B)$ draw an arc cutting the circular arc forming the path of B in the point B_1; from D with radius $(A\,B - D\,A)$ draw an arc cutting the path of B in B_2; then B_1 and B_2 will be the extreme positions of the pedal. The motion being in the direction of the arrow, and the speed of the machine being

uniform, the times taken by the pedal to perform its upward and downward movements are proportional to the lengths of the arcs A_1 9 A_2 and A_2 3 A_1. With the arrangement of the mechanism shown in the figure, the down-stroke takes a little less time than the up-stroke.

p p (fig. 501) is the polar curve of pedal speed, found by the method of section 32, and *k k*, the polar curve of speed of knee-joint, found by the method of section 321, for the dimensions of the gear marked in figure 497. The speed of the knee-joint is greatest when the crank is about 30° from its lowest position, then very rapidly diminishes to zero, and rapidly attains its maximum speed in the opposite direction. It should be remembered that the speed curve, *k k*, is obtained on the assumption that the ankle is

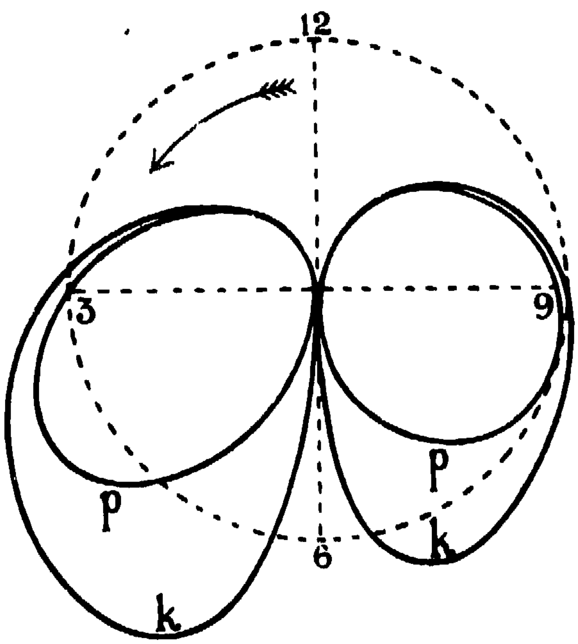


FIG. 501.

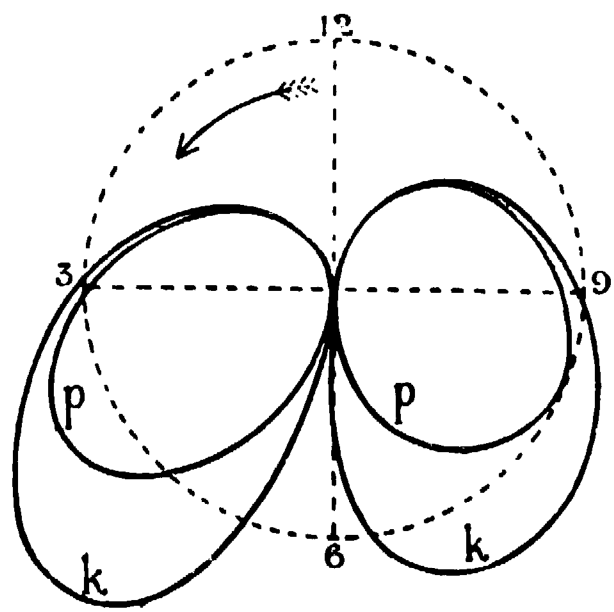


FIG. 502.

kept stiff during the motion. Using ankle action freely, the curve *k k* may not even approximately represent the actual speed of the knee; but the more rapid the variation of the radius-vectors to the curve *k k*, the greater will be the necessity for perfect ankle action. It should be noticed that with any mechanism a slight change in the position of the point *H* (fig. 497) may make a considerable change in the form of the curve *k k* (fig. 501).

In some of the early lever-and-crank geared tricycles the pedal was placed at the end of a lever which, together with the oscillating lever of the four-link kinematic chain, formed a bell

crank (see fig. 146). The treatment of the pedal motion in this case is the same as for the 'Facile' mechanism.

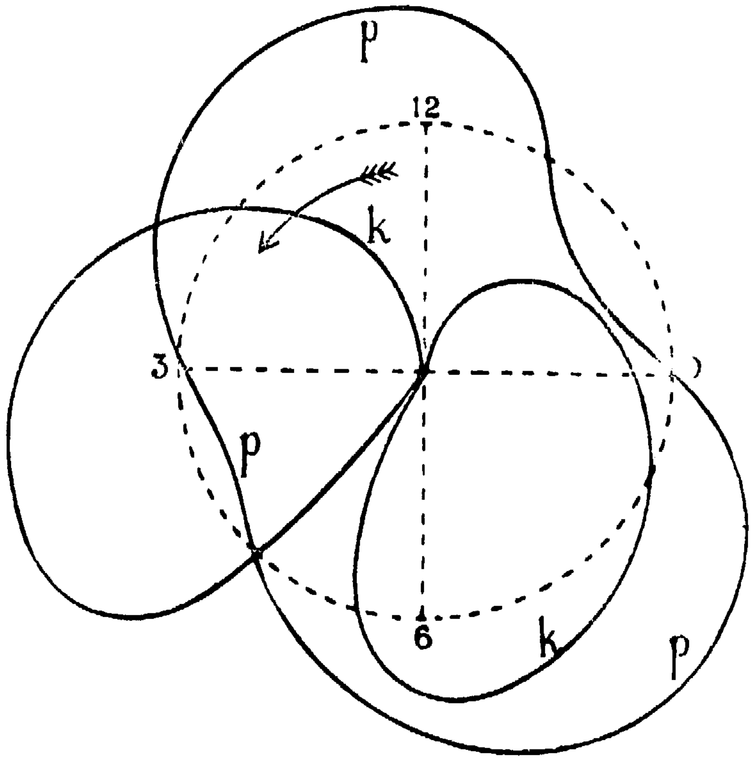


FIG. 503.

Geared Facile.—Figure 502 shows the polar curves of pedal speed, p p, and of speed of knee-joint, k k, for a Geared Facile:

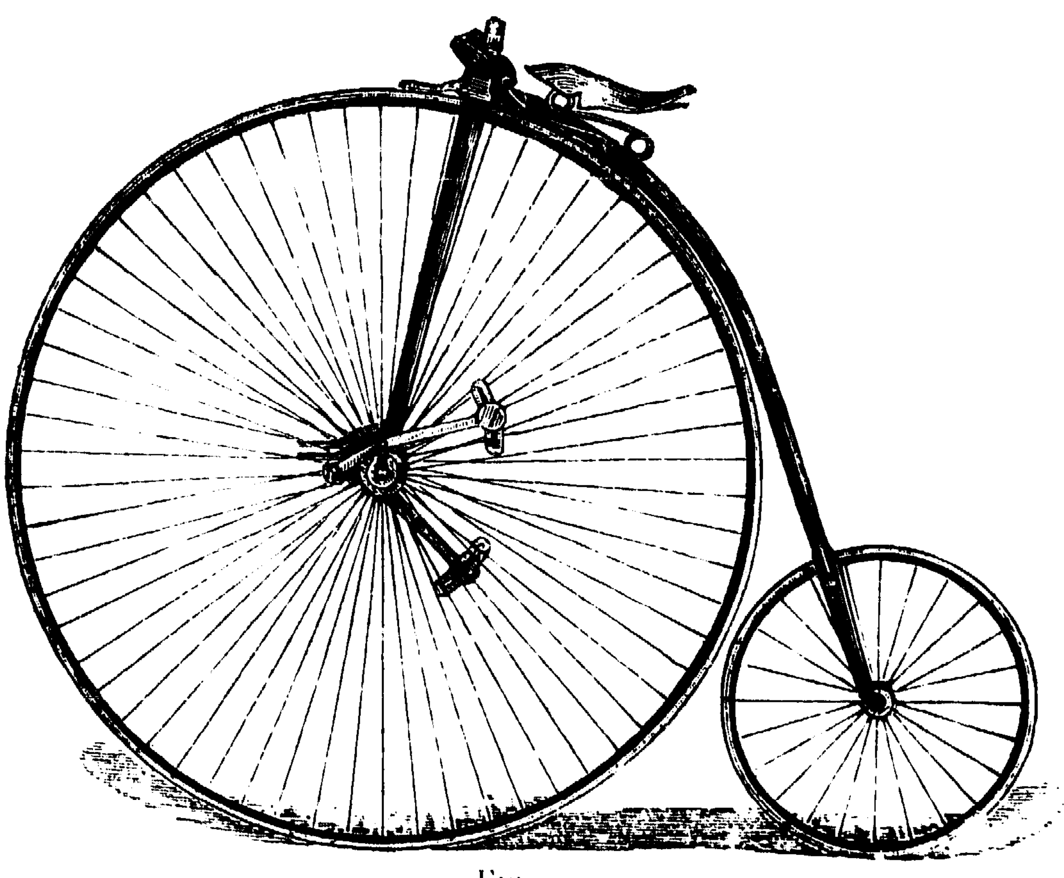


FIG. 504.

the dimensions of the mechanism being exactly the same as in figure 497, and the ratio of the diameters of the 'Sun-and-Planet' wheels being 2 : 1.

326. **The 'Xtraordinary'** was, perhaps, the first successful Safety bicycle, the driving mechanism being arranged so that the rider could use a large front wheel while sitting considerably further back and lower than was possible with an 'Ordinary.'

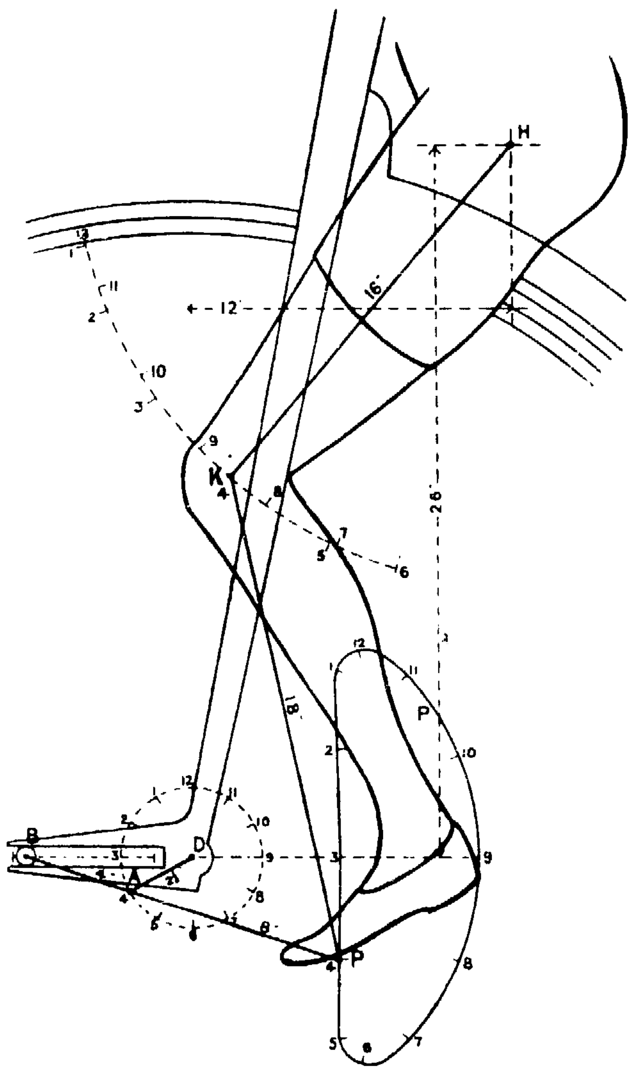


Fig. 503.

PP (fig. 498) is the pedal path in the 'Xtraordinary,' *p p* (fig. 503) the polar curve of pedal speed, and *k k* the polar curve of speed of the knee-joint. The down-stroke of the knee is performed much more quickly than the up-stroke, as is evident either from the polar speed curve, *k k*, or from the correspondingly numbered positions (fig. 498) of the knee and crank-pin. During the down-stroke of the knee, the crank-pin moves in the direction of the arrow, from 12 to 5; during the up-stroke, from 5 to 12.

327. **Claviger Bicycle.**—In the Claviger gear, as applied to the 'Ordinary' type of bicycle (fig. 504), the crank-pin was jointed to a lever, the front end of which moved, by means of a ball-bearing roller, along a straight slot projecting in front of the fork. At the rear end of the lever a segmental slot was formed to provide a vertical adjustment for the pedal, to suit riders of different heights. The mechanism is equivalent to the crank and connecting-rod of a steam-engine, the motion of the ball-bearing roller being the same as that of the piston or cross-head of the steam-engine. The mechanism may be derived from the four-link kinematic chain by

considering the radius of the arc in which the end, *B* (fig. 21), of the coupling-rod moves to be indefinitely increased. The con-

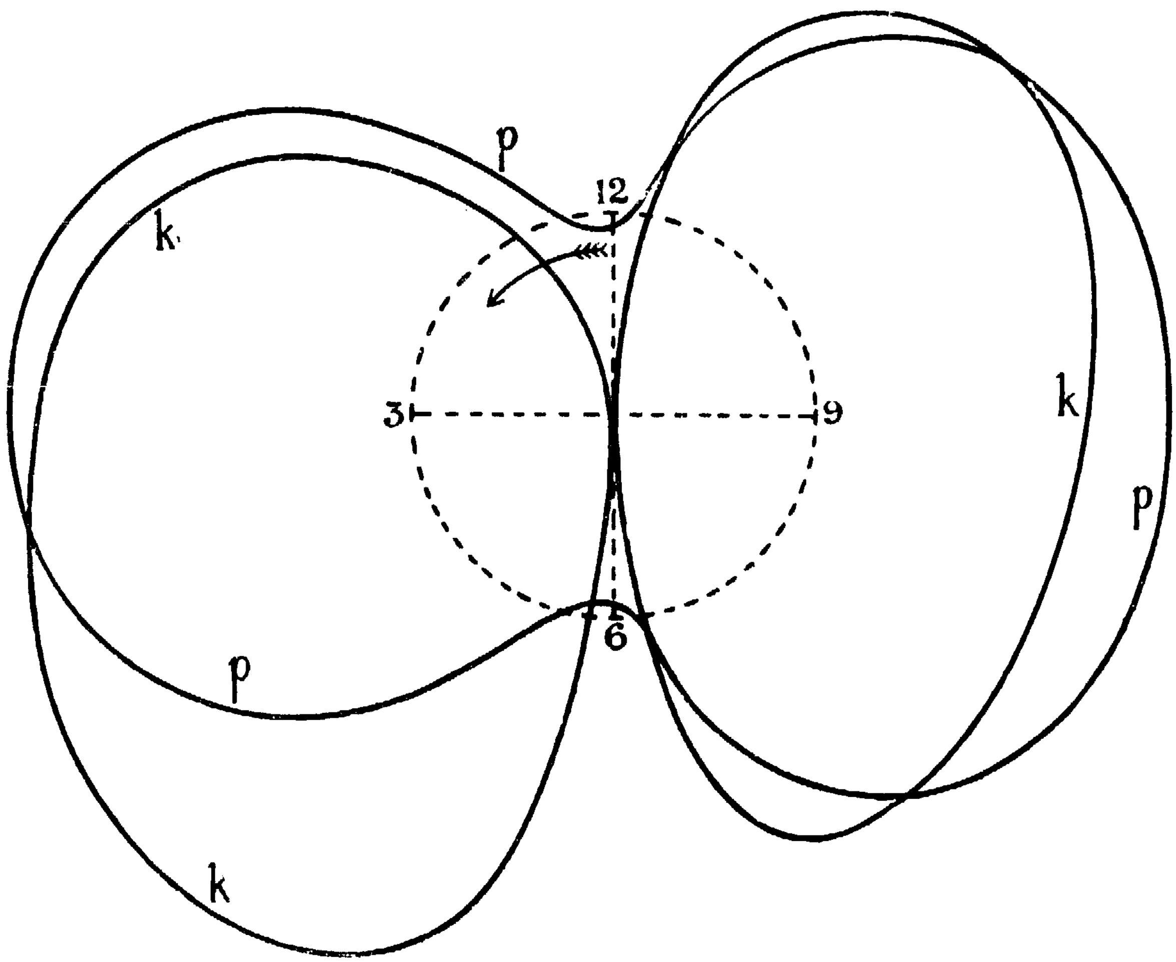


FIG. 506.

structions of figure 21 will be applicable, the only difference being that the straight line *B I* will always remain in the same direction,

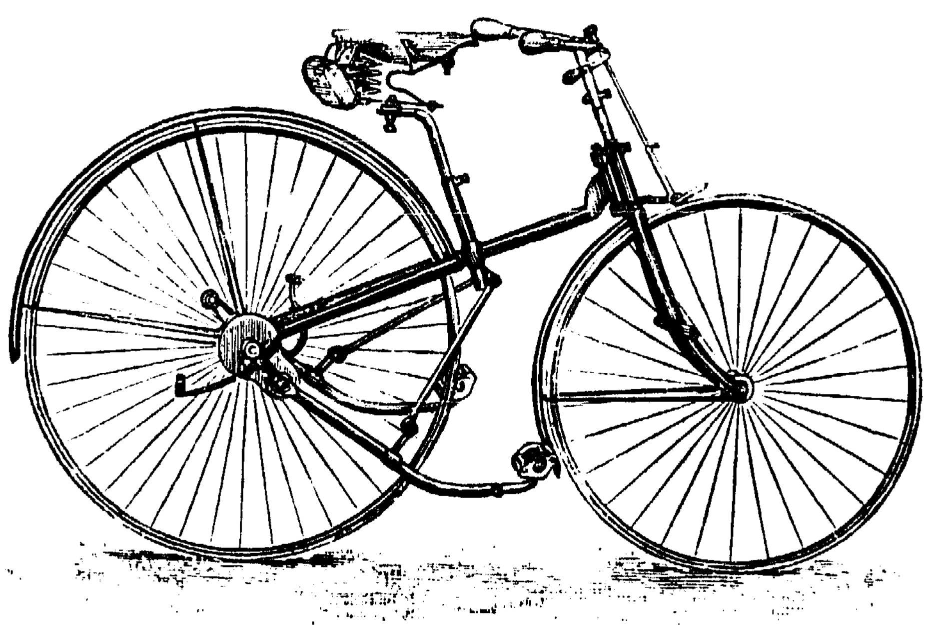


FIG. 507.

that is, at right angles to the straight slot. By bending the pedal lever downwards as shown (fig. 504), the position of the saddle is further backward and downward than in the 'Ordinary.'

P P (fig. 505) is the pedal path, *p p* (fig. 506) the polar curve of pedal speed, and *k k* the polar curve of speed of knee-joint, for the mechanism to the dimensions marked in figure 505.

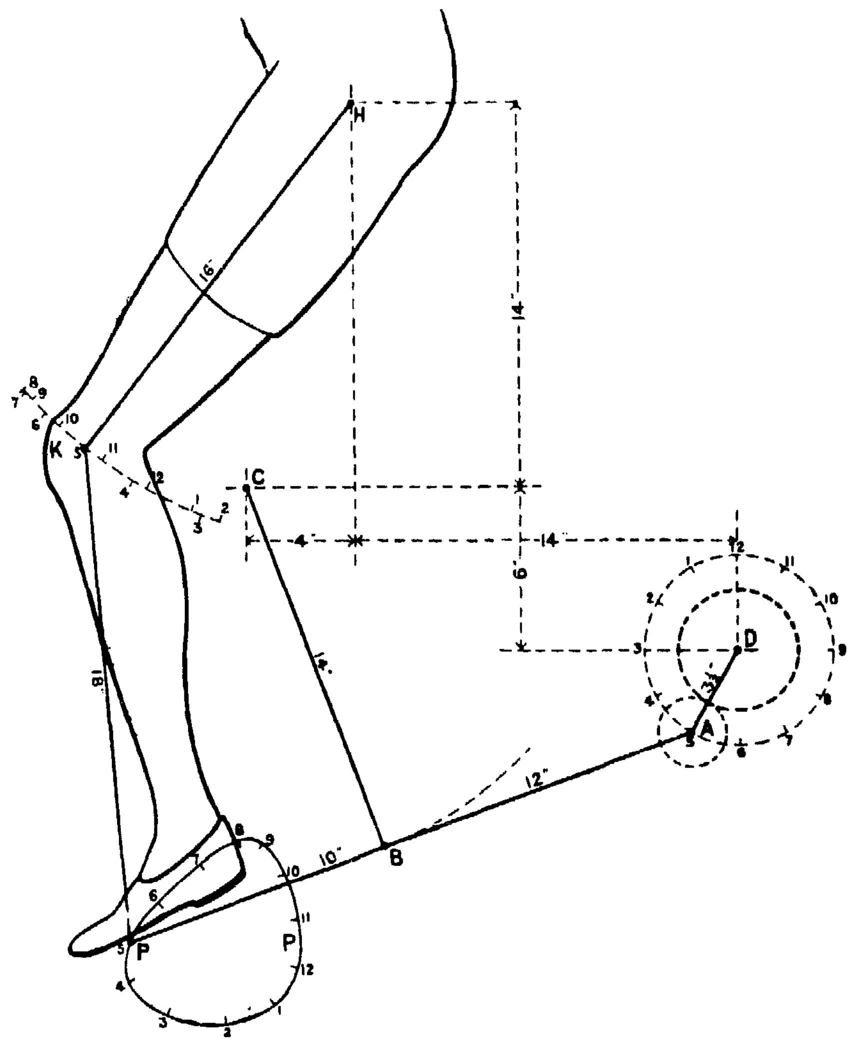


FIG. 508.

Geared Claviger.—*P P* (fig. 508) is the pedal path, *p p* (fig. 509) polar curve of pedal speed, and *k k* polar curve of speed of knee-joint, for a 'Geared Claviger' rear-driving Safety (fig. 507); the dimensions of the mechanism being as indicated in figure 508, and the ratio of the diameters of the 'Sun-and-Planet' wheels 2 : 1. The construction is as shown in figure 500.

A few peculiarities of the gear, as made to the dimensions marked in figure 508, may be noticed. The motion of the pedal in its oval path, is in the opposite direction to that of a pedal fixed to a crank. The speed of the pedal increases and diminishes three

times in each up-and-down stroke; the speed-curve, *p p* (fig. 509), shows this clearly. The pedal path (fig. 508) also indicates the same speed variation; the portions 2–3, 6–7, and 10–11, being

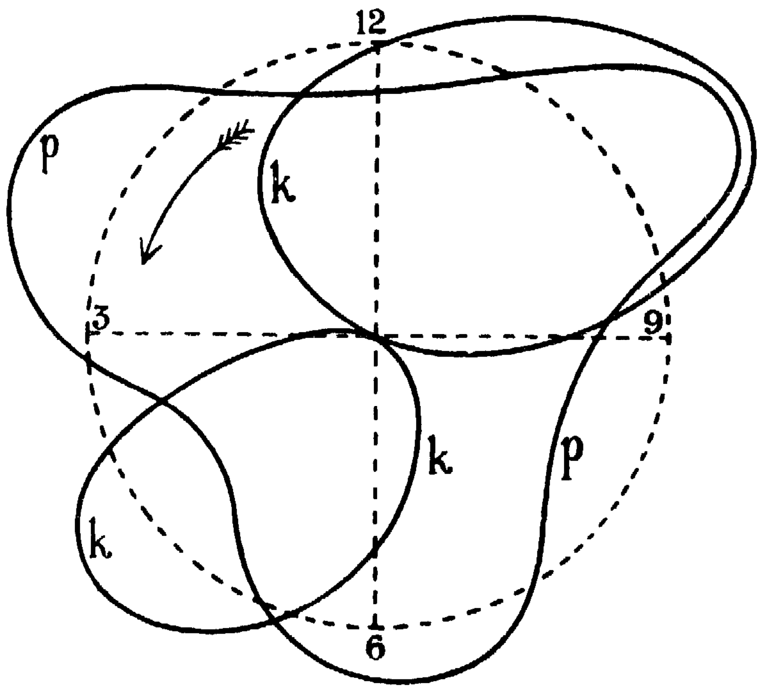


FIG. 509.

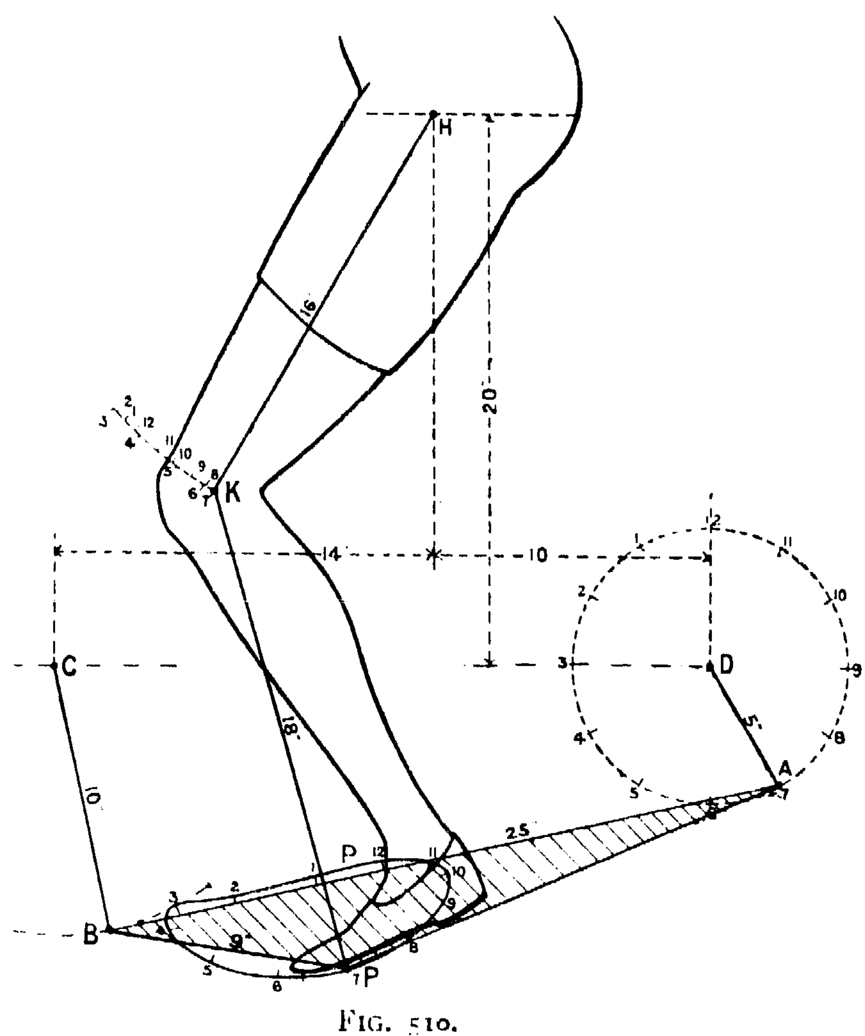


FIG. 510.

each longer than the adjacent portions, are passed over at greater speeds.

328. **Early Tricycles.**—In the *Dublin quadricycle* (fig. 117), and in some of the early lever-driven tricycles (fig. 142), the pedal was placed about the middle of the coupling-rod, one end of which was jointed to the crank-pin, the other to the end of the oscillating lever. The pedal path was an elongated oval, the vertical axis of which was shorter than the horizontal; the early designers aiming

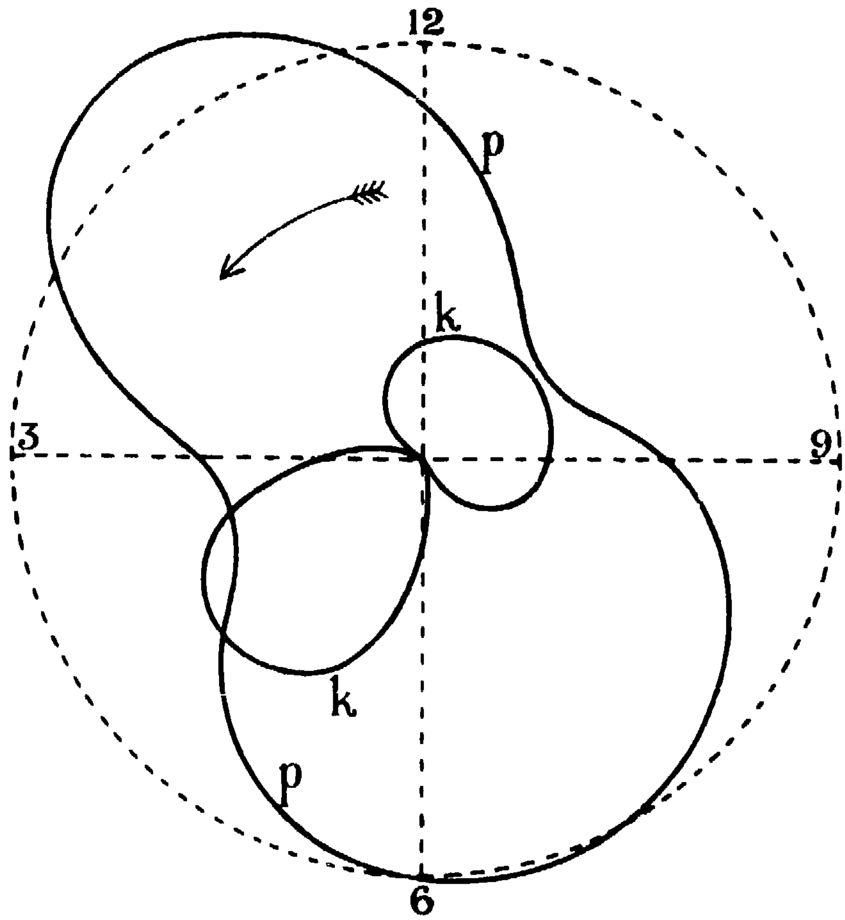


FIG. 511.

at giving the pedals a motion as nearly as possible like that of the foot during walking. *PP* (fig. 510) is the pedal path, *p p* (fig. 511) the polar curve of pedal speed, and *k k* the polar curve of speed of knee-joint, the dimensions of the mechanism being shown in figure 510. The construction is as shown in figure 498.

It may be noticed either from figure 510 or the curve *k k* (fig. 511) that the down-stroke of the knee is performed in one-third the time of one revolution of the crank, the up-stroke in two-thirds. Also, the knee is at the top of its stroke, when the crank is nearing the horizontal position, descending.

CHAPTER XXIX

TYRES

329. **The Tyre** is that outer portion of the wheel which actually touches the ground. The tyres of most road and railway vehicles are of iron or steel, and in the early days of the bicycle, when wooden wheels were used, their tyres were also of iron. The tyre of a wooden wheel serves the double purpose of keeping the component parts of the wheel in place, and providing a suitable wearing surface for rolling on the ground.

330. **Rolling Resistance on Smooth Surfaces.**—The rolling friction of a wheel on a smooth surface is small, and if the surfaces of the tyre and of the ground be hard and elastic the rolling friction, or tyre friction, may be neglected in comparison with the friction of the wheel bearings. This is the case with railway wagons and carriages. A short investigation of the nature of rolling friction has been given in section 78.

In Professor Osborne Reynolds' experiments the rolling took place at a slow speed. When the speed is great another factor must be considered. The tyre of a circular wheel rolling on a flat surface gets flattened out, and the mutual pressure is distributed over a surface. Let c (fig. 512) be the geometrical point of contact, a_1 and a_2 two points at equal distances in front of, and behind, c; p_1 and p_2 the intensities of the pressures at a_1 and a_2 respectively. The pressure p_1 opposes, the pressure p_2 assists, the rolling of the wheel. If the rolling takes place slowly, it is possible that p_2 may be equal to p_1, and the resultant reaction on the wheel

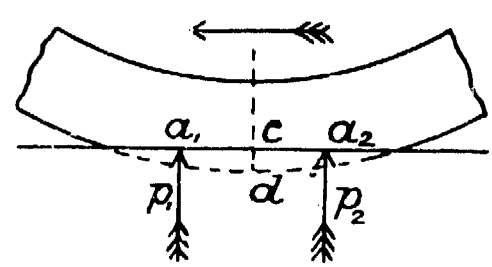


FIG. 512.

may pass through the centre. But in all reversible dynamical actions which take place quickly, it is found that there is a loss of energy, which varies with the quickness of the action. The term 'hysteresis,' first used by Professor Ewing in explaining the phenomenon as exhibited in the magnetisation of iron, may be used for the general phenomena. In unloading a spring quickly, the load corresponding to a given deformation is less than when loading it; more work is required to load the spring than it gives out during the removal of the load. If $O\ a\ P$ (fig. 513) be the stress-strain curve during loading, that during unloading will be $P\ b\ O$, and the area $O\ a\ P\ b\ O$ will be the energy lost by hysteresis. Thus, p_2 is less than p_1, the ratio $\frac{p_2}{p_1}$ lying between 1 and e, the index of elasticity.

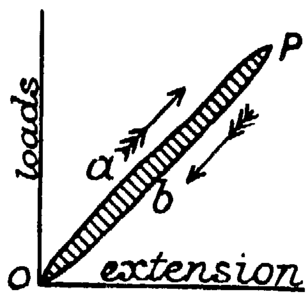


FIG. 513.

p_1, varies with the distance of a_1 from c, and is a maximum when a_1 coincides with c. Assuming the ratio $\frac{p_2}{p_1}$ to remain constant for all positions of a_1 and a_2 relative to c, we may say that the energy lost is proportional to

$$\frac{(p_1 - p_2)}{p_1} \cdot \overline{c\,d},$$

$\overline{c\,d}$ being the radial displacement of a point on the tread of the tyre.

Comparing three tyres of rubber, air, and steel respectively rolling on a *perfectly hard* surface, $\frac{p_1 - p_2}{p_1}$ will possibly be smallest for air, and largest for rubber; while the displacement $\overline{c\,d}$ will be smallest for steel. The rolling resistance of the steel tyre will be least, that of the rubber tyre greatest.

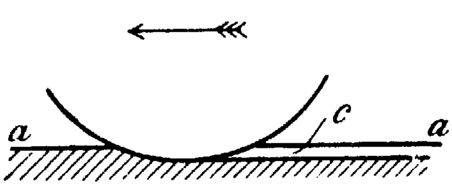


FIG. 514.

331. **Metal Tyre on Soft Road.**—The road surfaces over which cycles have to be propelled are not always hard and elastic, but are often quite the opposite. If a hard

metal tyre be driven over a soft road *a a* (fig. 514) it sinks into it and leaves a groove *c* of quite measurable depth. The resistance experienced in driving a cycle with narrow tyres over a soft road is mainly due to the work spent in forming this groove.

332. **Loss of Energy by Vibration.**—The energy lost on account of the impact of the tyre on the ground is proportional to the total mass which partakes of the motion of impact (see chap. xix.). In a rigid wheel with rigid tyres, this will consist of the whole of the wheel, and of that part of the frame which may be rigidly connected to, and rest on, the spindle of the wheel. If no saddle springs be used, part of the mass of the rider will also be included. The energy lost by impact, and which is dissipated in jar on the wheel of the machine, must be supplied by the motive power of the rider; consequently any diminution of the energy dissipated in shock, will mean increased ease of propulsion of the machine.

The state of the road surface is a matter generally beyond the control of the cyclist or cycle manufacturer, and therefore so also are the velocities of the successive impacts that take place. However, the other factor entering into the energy dissipated, the mass m rigidly connected with the tyre is under the control of the cycle makers. In the first bicycles made with wooden wheels and iron tyres, and sometimes without even a spring to the seat, the mass m included the whole of the wheel and a considerable proportion of the mass of the frame and rider; so that the energy lost in shock formed by far the greatest item in the work to be supplied by the rider. The first improvement in a road vehicle is to insert springs between the wheel and the frame. This practically means that the up and down motion of the wheel is performed to a certain extent independently of that of the vehicle and its occupants; the mass m in equation (2), chapter xix., is thus practically reduced to that of the wheel. The effort required to propel a spring vehicle along a common road is much less than that for a springless vehicle.

333. **Rubber Tyres.**—If the tyre of the wheel be made elastic so that it can change shape sufficiently during passage over an obstacle, the motion of the wheel centre may not be perceptibly

affected, and the mass subjected to impact may be reduced to that of a small portion of the tyre in the neighbourhood of the point of contact. Thus, the use of rubber tyres on an ordinary road greatly reduces the amount of energy wasted in jar of the machine. Again, the rubber tyre being elastic, instead of sinking into a moderately soft road, is flattened out. The area of contact with the ground being much larger, the pressure per unit area is less, and the depth of the groove made is smaller; the energy lost by the wheel sinking into the road is therefore greatly reduced by the use of a rubber tyre.

Rolling Resistance of Rubber Tyres.—The resistance to rolling of a rubber tyre is of the same nature as that discussed in section 78, but the amount of compression of the tyre in contact with the ground being much greater than in the case of a metal wheel on a metal rail, the rolling resistance is also greater. This may appear startling to cyclists, but this slight disadvantage of rubber as compared with steel tyres is more than compensated by the yielding quality of the rubber, which practically neutralises the minor inequalities of the road surface.

334. **Pneumatic Tyres in General.**—The good qualities of a rubber tyre, as compared with a metal tyre for bicycles, are present to a still greater degree in pneumatic tyres. In a $\frac{3}{4}$-inch rubber tyre, half of which is usually buried in the rim of the wheel, the maximum height of a stone that can be passed over without influencing the motion of the wheel as a whole, cannot be much greater than a quarter of an inch. With a 2-inch pneumatic tyre, most of which lies outside the rim, a stone 1 inch high may be passed over without influencing the motion of the wheel to any great extent, provided the speed is great. The provision against loss of energy by impact in moving over a rough road is more perfect in this case. Again, the tyre being of larger diameter, its surface of contact with the ground is greater, and the energy lost by sinking into a road of moderate hardness is practically *nil.*

Rolling Resistance of Pneumatic Tyres.—Considering the tyre as a whole to be made of the material 'air,' and applying the result of section 194, if the material be perfectly elastic, there would

be absolutely no rolling resistance. Now for all practical purposes air may be considered perfectly elastic, and there will be no dissipation of energy by the air of the tyre. The indiarubber tube in which the air is confined, and the outer-cover of the tyre, are, however, made of materials which are by no means perfectly elastic. The work done in bending the forward part of the cover will be a little greater than that restored by the cover as it regains its original shape. Probably the only appreciable resistance of a pneumatic tyre is due to the difference of these two forces. The work expended in bending the tyre will be greater, the greater the angle through which it is bent. This angle is least when the tyre is pumped up hardest; and therefore on a smooth racing track pneumatic tyres should be pumped up as hard as possible.

Again, the work required to bend the cover through a given angle will depend on its stiffness; in other words, on its moment of resistance to bending. For a tyre of given thickness d this resistance will be greatest when the tyre is of the single-tube type, and other things being equal, will be proportional to the square of the thickness d. If the cover could be made of n layers free to slide on each other, each of thickness $\frac{d}{n}$, the resistance of each layer to bending would be proportional to $\frac{d^2}{n^2}$, and that of the n layers constituting the complete covering to $\frac{d^2}{n}$. Thus for a tyre of given thickness its resistance is inversely proportional to the number of separate layers composing the cover. This explains why a single-tube tyre is slower than one with a separate inner air-tube; it also explains why racing tyres are made with the outer-cover as thin as possible.

Relation between Air Pressure and Weight Supported.—Let a pneumatic tyre subjected to air pressure p support a weight W. The part of the tyre near the ground will be flattened, as shown in figure 515. Let A be the area of contact with the ground, and let q be the average pressure per square inch on the ground. Then, if we assume that the tyre fabric is perfectly flexible, since the part in contact with the ground is quite flat, the

pressures on the opposite sides must be equal. Therefore $q = p$. But the only external forces acting on the wheel are W and

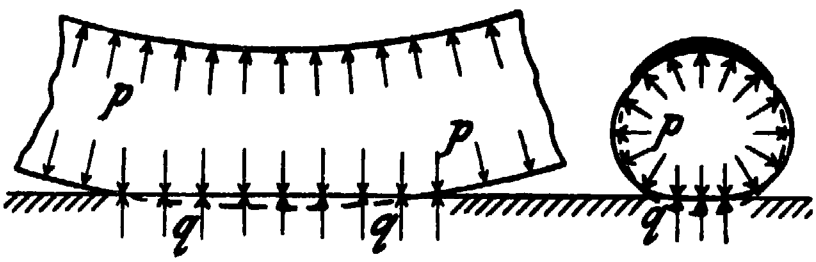


FIG. 515.

the reaction of the ground. These must be equal and opposite, therefore

$$A p = W \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Let p_0 and V_0 respectively be the pressure per sq. in., and the volume of air inside the tyre, before the weight comes on the wheel; and let p and V be these quantities when the tyre is deformed under the weight. The air is slightly compressed; *i.e.* V is slightly less than V_0, and p is a little greater than p_0. Now the pressure of a given quantity of gas is inversely proportional to the volume it occupies; *i.e.*

$$\frac{p}{p_0} = \frac{V_0}{V} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

p and p_0 being absolute pressures.

Example.—If a weight of 120 lbs. be carried by the driving-wheel of a bicycle, and the pneumatic tyre while supporting the load be pumped to an air pressure of 30 lbs. per sq. in. above atmosphere, the area of contact with the ground $= \frac{120}{30}$ $= 4$ sq. in.

If the diameter of the wheel be 28 inches and that of the inner tube be $1\frac{3}{4}$-inch, it would be easy by a method of trial and error to find a plane section of the annulus having the area required, 4 sq. in. If we assume that the part of the tyre not in contact with the ground retains its original form, which is strictly true except for the sides above the part in contact with the ground, the diminution of the volume of air inside the tyre would be the volume cut off by this plane section. In the above example this

decrease is less than 1 cubic inch. The original volume of air is equal to the sectional area of the inner tube multiplied by its mean circumference. The area of a $1\frac{3}{4}$-inch circle is 2·405 sq. in., the circumference of a circle 26 inches diameter is 81·68 ins.

$$\therefore \; V_0 = 2{\cdot}405 \times 81{\cdot}68 = 196{\cdot}5 \text{ cubic inches.}$$

V may be taken 195·5 cubic inches. Taking the atmospheric pressure at 14·7 lbs. per sq. in., $p = 30 + 14{\cdot}7 = 44{\cdot}7$. Hence, substituting in (2)

$$p_0 = \frac{44{\cdot}7 \times 195{\cdot}5}{196{\cdot}5} = 44{\cdot}47 \text{ lbs. per sq. in. absolute}$$
$$= 29{\cdot}77 \text{ lbs. per sq. in. above atmosphere;}$$

and therefore the pressure of the air inside the tyre has been increased by 0·23 lb. per sq. in.

335. **Air-tube.**—The principal function of the air-tube is to form an air-tight vessel in which the air under pressure may be retained. It should be as thin and as flexible as possible, consistent with the necessity of resisting wear caused by slight chafing action against the outer-cover. It should also be slightly extensible, so as to adapt itself under the air-pressure to the exact form of the rim and outer-cover. Indiarubber is the only material that has been used for the air-tube.

Two varieties of air-tubes are in use: the continuous tube and the butt-ended tube. The latter can be removed from a complete outer tube by a hole a few inches in length, while the former can only be removed if the outer-cover is in the form of a band with two distinct edges.

336. **Outer-cover.**—The outer-cover has a variety of functions to perform. Firstly, it must be sufficiently strong transversely and longitudinally to resist the air-pressure. Secondly, in a driving-wheel it must be strong enough to transmit the tangential effort from the rim of the wheel to the ground. Thirdly, the tread of the tyre should be thick enough to stand the wear and tear of riding on the road, and to protect the air-tube from puncture. Fourthly, though offering great resistance to elongation by direct tension, it should be as flexible as possible, offering very

little resistance to bending as it comes into, and leaves, contact with the ground, and as it passes over a stone.

Stress on Fabric.—We have already investigated (sec. 84) the tensile stress on a longitudinal section of a pneumatic tyre. We will now investigate that on a transverse section. Consider a transverse section by a plane passing through the axis of the wheel, and therefore cutting the rim at two places. The upper part of the tyre is under the action of the internal pressure, and the pull of the lower portion at the two sections. If we imagine the cut ends of the half-tyre to be stopped by flat plates, it is evident that the resultant pressure on the curved portion of the half-tyre will be equal and opposite to the resultant pressure on the flat ends. If d and t be respectively the diameter and thickness of the outer-cover, and p be the air-pressure, the area of each of the flat ends is $\frac{\pi d^2}{4}$, and therefore the resultant pressure on the curved surface is $2\frac{\pi}{4}d^2 p$.

The area of the two transverse sections of the outer-cover is $2\pi d t$; therefore the stress on the transverse section is

$$f = \frac{2\frac{\pi}{4}d^2 p}{2\pi d t} = \frac{p d}{4 t} \quad \ldots \ldots \quad (3)$$

Comparing with section 84, the stress on a transverse section of the fabric is half that on a longitudinal section.

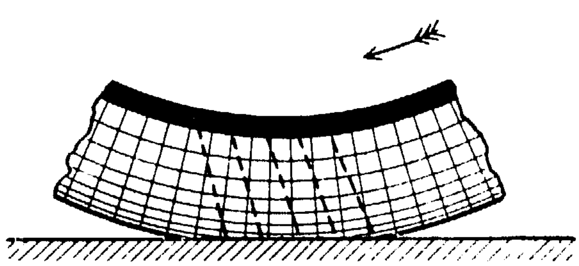


FIG. 516.

Spiral Fibres.—The first pneumatic tyres were made with canvas having the fibres running transversely and circumferentially (fig. 516). The fibres of a woven fabric, intermeshing with each other, are not quite straight, and offer resistance to bending as it comes into and leaves contact with the ground. Further, when the fibres are disposed transversely and circumferentially the cover cannot transmit any driving effort from the rim of the wheel to the

ground, until it has been distorted through a considerable angle, as shown by the dotted lines.

In the 'Palmer' tyre the fabric is made up of parallel fibres embedded in a thin layer of indiarubber, the fibres being wound

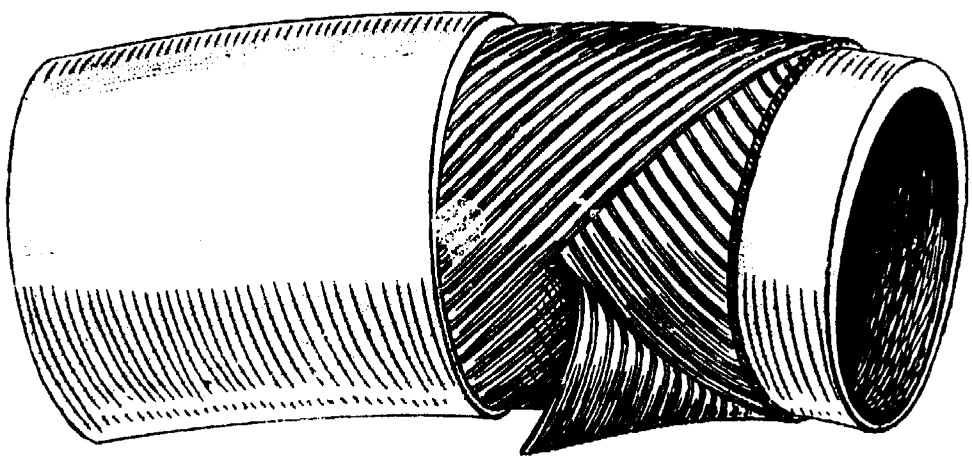


FIG. 517.

spirally (fig. 517) round an inner tube. Two layers of this fabric are used, the two sets of spirals being oppositely directed. When a driving effort is being exerted, the portion of the tyre between the ground and the rim is subjected to a shear parallel to the ground, which is, of course, accompanied by a shear on a vertical plane. This shearing stress is equivalent to a tensile stress in the direction *c c* (fig. 518), and a compressive stress in the direction *d d* (see sec. 105); consequently the fabric with spiral fibres is much better able to transmit the driving effort from the rim to the ground. This construction is undoubtedly the best for driving-wheel tyres; but in a non-driving wheel practically no tangential or shearing stress is exerted on the fabric of the tyre. Therefore, for a non-driving wheel the best arrangement is, possibly, to have the fibres running transversely and longitudinally; the brake should then be applied only to the driving-wheel.

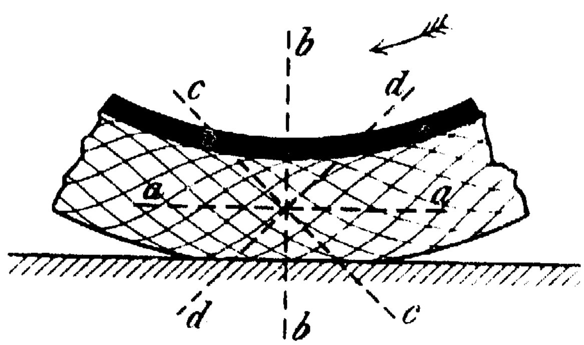


FIG. 518.

The tyre with spirally arranged fibres has another curious

property. It has been shown that the tensile stress on the transverse section *b b* of the tyre is half that on the longitudinal section *a a*. Let the stress on the section *b b* be denoted by p, that on *a a* by $2p$. This state of stress is equivalent to two simultaneously acting states of stress: the first, equal tensile stresses $\frac{3p}{2}$ on both sections; the second, a tension $\frac{p}{2}$ on *a a*, and a compression $\frac{p}{2}$ on *b b*. The first system of stress tends to stretch the fibre equally in all directions; the second state of stress is equivalent to shearing stresses $\frac{p}{2}$ on the planes *c c* and *d d* parallel to the spiral fibres. If the tyre be inflated free from the rim of the wheel, the fabric cannot resist the distortion due to this shearing stress, so that the tension $\frac{p}{2}$ on the section *a a* tends to increase the size of the transverse section of the tyre, and the compression $\frac{p}{2}$ on *b b* tends to shorten the circumference of the tyre. Thus, finally, the act of inflation tends to tighten the tyre on the rim.

337. **Classification of Pneumatic Tyres.**—Pneumatic tyres have been subdivided into two great classes: Single-tube tyres, in which an endless tube is made air-tight, and sufficiently strong to resist the air-pressure; Compound tyres, consisting of two parts—an inner air-tube and an outer-cover. Quite recently, a new type, the 'Fleuss' tubeless tyre, has appeared. Mr. Henry Sturmey, in an article on 'Pneumatic Tyres' in the 'Cyclist's Year Book' for 1894, divides compound tyres into five classes, according to the mode of adjustment of the outer-cover to the rim, viz.: Solutioned tyres, Wired tyres, Interlocking and Inflation-held tyres, Laced tyres, and Band-held tyres.

A better classification, which does not differ essentially from the above, seems to be into three classes, taking account of the method of forming the chamber containing the compressed air, as follows:

Class I., with complete tubular outer-covers. This would include all single-tube tyres, most solutioned tyres, and some

laced tyres. Tyres of this class can be inflated when detached from the rim of the wheel; in fact, the rim is not an integral part of the tyre, as in the two following classes. This class may be referred to as *Tubular* tyres.

Class II., in which the transverse tension on the outer-cover is transmitted to the edges of the rim, so that the outer-cover and rim form one continuous tubular ring subjected to internal air-pressure. The 'Clincher' tyre is the typical representative of this class. With most tyres of this class the compression on the rim due to the pull of the spokes is reduced on inflation. This class will be referred to as *Interlocking* tyres.

Class III., in which the transverse tension on the outer-cover is transmitted to the edges of the latter, and there resisted by the longitudinal tension of wires embedded in the cover. This class includes most wired tyres. With tyres of this class the initial compression on the rim is increased on inflation. This class will be referred to as *Wired* tyres, and may be subdivided into two sections, according as the wire is endless, or provided with means for bringing the two ends together, and so adjusting the wire on the rim.

338. **Tubular Tyres.**—*Single-tube tyres*, which form an important group in this class, are made up of an outer layer of rubber forming the tread which comes in contact with the ground, a middle layer of canvas, or other suitable material, to provide the necessary strength and inextensibility, and an inner air-tight layer of rubber. The 'Boothroyd' and the 'Silvertown' were among the most successful of these single-tube tyres. The 'Palmer' tyre (fig. 517) was originally made as a single-tube.

Since a solid plate of given thickness offers more resistance to bending than two separate plates having the same total thickness, the resilience of a tyre is decreased by cementing the air-tube and outer-cover together.

Solutioned tyres.—The original 'Dunlop' tyre (fig. 519), which was the originator of the principle of air tyres for cycles, belongs to this class. The outer-cover consists of a thick tread of rubber *A* solutioned to a canvas strip *B*. A complete woven tube of canvas *H*, encircles the air-tube *C*, and is solutioned to the rim *E*, which is previously wrapped round by a canvas strip *D*; while

the flaps of the outer-cover are solutioned to the inner surface of the rim, one flap being lapped over the other, the side being slit to pass the spokes. A strip of canvas *F*, solutioned over the flaps, makes a neat finish.

In the Morgan and Wright tyre, the air-tube is butt-ended, or rather scarf-ended, the two ends overlapping each other about

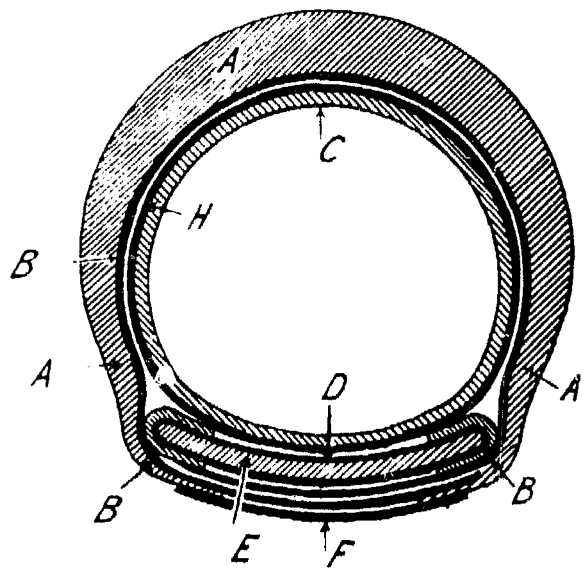


FIG. 519.

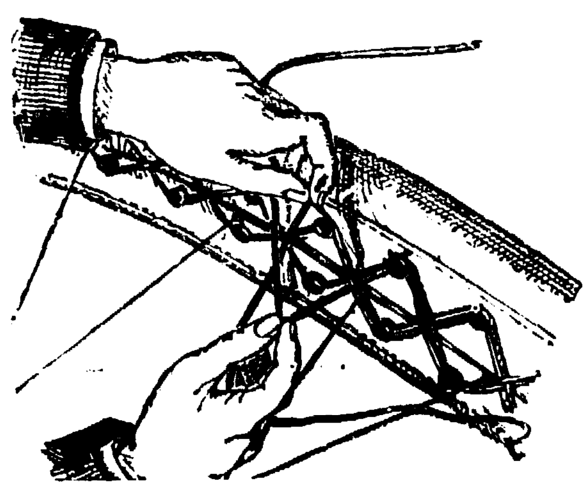


FIG. 520.

eight or ten inches. The outer-cover forms practically a tube slit for a few inches along its under side; this opening serves for the insertion of the air-tube, and is laced up when the air-tube is in place. When partially inflated the tyre is cemented on to the rim.

Laced tyres.—In Smith's 'Balloon' tyre (fig. 520) the outer-cover was furnished with stud hooks at its edges, and enveloped the rim completely; its two edges were then laced together.

339. **Interlocking Tyres.**—In this class of tyres the circumferential tension near the edge of the outer-cover is transmitted direct to the rim of the wheel, by suitably formed ridges, which on inflation are forced into and held in corresponding recesses of the rim.

Inflation-held Tyres.—In tyres which depend primarily on inflation for the fastening to the rim, the edge of the outer-cover is continued inwards forming a toe beyond the ridge or heel, the

air-pressure on the toe keeping the heel of the outer-cover in close contact with the recess of the rim.

The 'Clincher' tyre was the first of this type. The 'Palmer' detachable tyre (fig. 521), so far as regards the fastening of the outer-cover to the rim, is identical with the 'Clincher.'

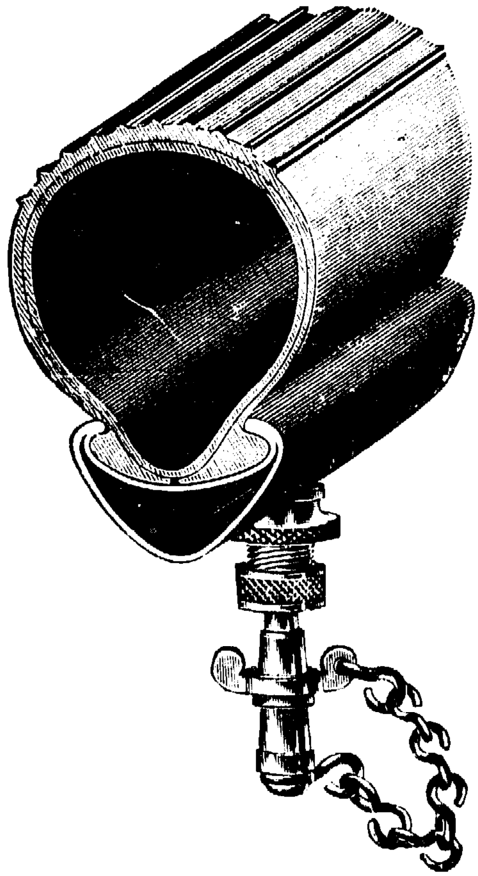


FIG. 521.

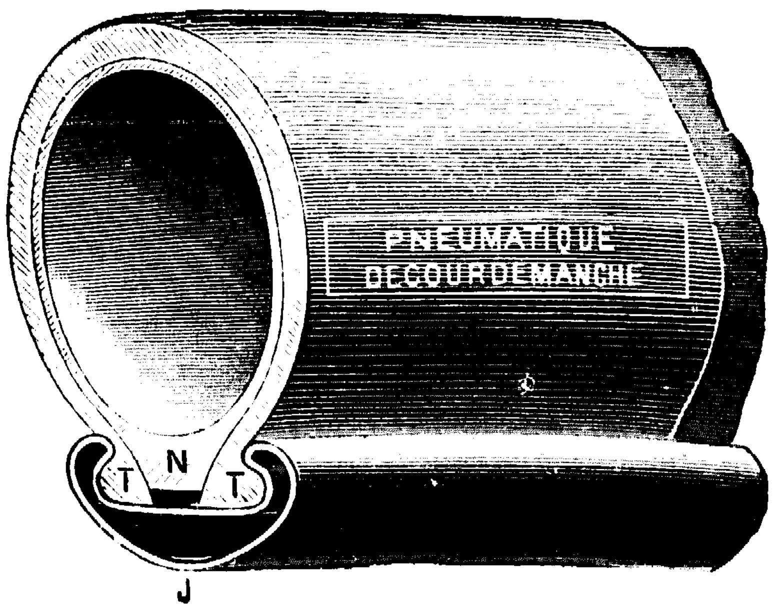


FIG. 522.

The 'Decourdemanche' tyre (fig. 522) is of the 'Clincher' type, but it has a wedge thickening *N* on the inner part of the air-tube, which on inflation is pressed between the ridges *T* of the outer-cover, and forces them into the recesses of the rim.

The 'Swiftsure' tyre differs essentially from those previously described. The outer-cover is furnished at the edges with circular ridges which lie in a central deep narrow mouthed groove of the rim. The mouth of the groove is just large enough to admit the ridge of the cover, while the body of the groove is wide enough to let them lie side by side. On inflation, the tendency is to draw both ridges from the groove together, so that they lock each other at the mouth, and thus the tyre is held on the rim.

Hook-tyres.—In this subdivision the positive fastening of the outer-cover to the rim does not depend merely on inflation; but the pull of the cover can be transmitted to the rim in the proper direction, even though there be no pressure in the air-tube.

In the original 'Preston-Davies' tyre eye-holes were formed near the edges of the outer-cover; these were threaded on hooks turned slightly inwards, so that on inflation the cover was held securely to the rim.

The 'Grappler' tyre is a successful modern example of this same class. Near each edge of the outer-cover a series of turned-back hooks or *grapplers* are fastened. These engage with the in-turned edge of the rim, so that on inflation the tyre is securely fastened.

Band-held tyres.—In the 'Humber' pneumatic tyre (fig. 523) the outer-cover *A* is held down on the rim *D* by means of a locking plate *C* on which the air-tube *B* rests.

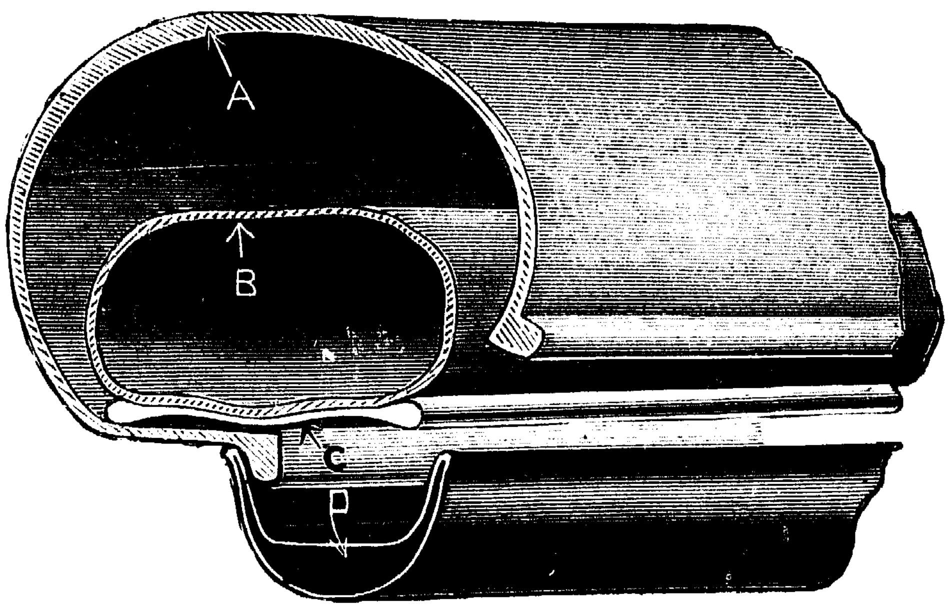


FIG. 523.

In the 'Woodley' tyre (fig. 524) it is possible that the flap acts in somewhat the same way as the plate in the 'Humber' tyre.

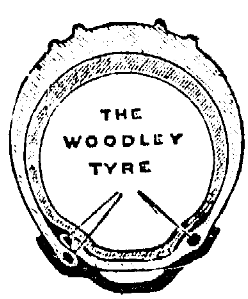


FIG. 524.

The 'Fleuss' tubeless tyre (fig. 525) is fixed to the rim on the 'Clincher' principle. The inner surface of the tyre is made air-tight, and thus a separate air-tube is dispensed with. A flap, permanently fastened to one edge of the tyre, is pressed on the other edge, when inflation is completed. The difficulty of keeping an air-tight joint between this loose flap and the edge of the tyre, right round the circumference (a length of over six feet), has been successfully overcome.

340. **Wire-held Tyres.**—The mode of fastening to the rim, used in this class of pneumatic tyre, differs essentially from that used in the other classes. Wires W (fig. 526) are embedded near the edges of the outer-cover C. On infla-

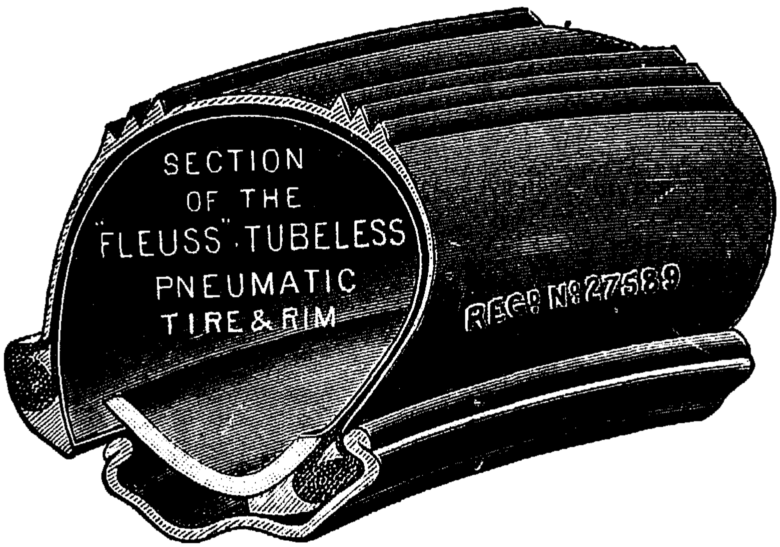


FIG. 525.

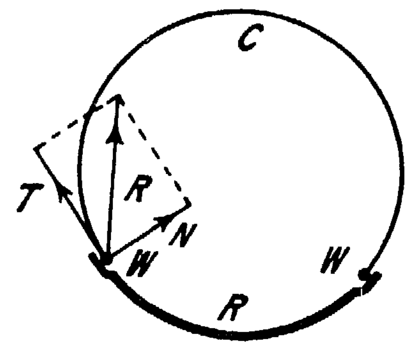


FIG. 526.

tion, a transverse tension T is exerted on the outer-cover, and transmitted to the wire W, tending to pull it out of the rim R. The wire is also pressed against the rim, the reaction from which N is at right angles to the surface. The resultant R of the forces T and N must lie in the plane of the wire W, and constitutes a radial outward force acting at all points of the ring formed by the wire. Thus, the chamber containing the air under pressure is formed of two portions : the outer-cover, subjected to tension T; and the rim R subjected to bending by the pressures N exerted by the wires W.

Let d be the diameter of the air-tube (not shown in figure 526), D the diameter of the ring formed by the wire W, and p the air-pressure. Then, by (7) chap. x., the force T per inch length of the wire is $\frac{p\,d}{2}$. The force R will be greater than T, depending on the angle between them. In the 'Dunlop' detachable tyre, this angle is about 30°, and therefore $R = 1{\cdot}155\ T$. The longitudinal pull P on the wire W is, by another application of the same formula,

$$P = \frac{1{\cdot}155\ T D}{2} = {\cdot}289\, p\, d\, D \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Example.—A pneumatic tyre with air-tube $1\frac{3}{4}$ in. diameter is fixed by wires forming rings 24 ins. diameter; and has an air pressure of 30 lbs. per sq. in.; the pull on each wire is therefore $\cdot289 \times 30 \times 1\cdot75 \times 24 = 364$ lbs.

If the wire be No. 14 W. G. its sectional area (Table XII.), p. 346, is ·00503 sq. in., and the tensile stress is

$$\frac{364}{503} = 72300 \text{ lbs. per sq. in.}$$
$$\text{or } 32\cdot3 \text{ tons per sq. in.}$$

Wire-held tyres may be sub-divided into two classes; in one the wire is in the form of an endless ring, and is therefore non-adjustable, in the other the ends of the wire are fastened by suitable mechanism, so that it can be tightened or released at pleasure.

The 'Dunlop' Detachable Tyre (fig. 527) is the principal representative of the endless wired division. In it two endless wires are embedded near the edges of the outer-cover. These wires form rings of less diameter than the extreme diameter of the rim, and are lodged in suitable recesses of the rim. The rim is deeper at the middle than at the recesses for the wire. To detach the tyre, after deflation, one part of one edge of the outer-cover is depressed into the bottom of the rim, the opposite part of the same edge will be just able to surmount the rim, and one part of the wire being got outside the rest will soon follow.

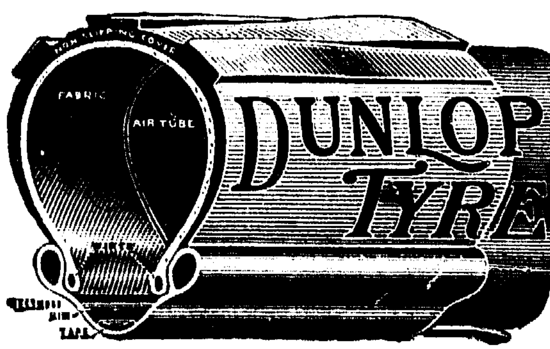


FIG. 527.

The 'Woodley' tyre (fig. 524) is formed from the 'Dunlop' by adding a flap to the outer-cover, this flap extending from one of the main fixing wires to the other, and so protecting the air-tube from contact with the rim.

In the original 'Beeston' tyre this flap was extended so far as to completely envelop the air-tube. In the newer patterns this wrapping has been discarded, and the 'Beeston' is practically the same as the 'Dunlop' detachable.

The '1895 Speed' tyre, made by the Preston-Davies Valve and Tyre Company, is fixed to the rim by means of a continuous wire of three coils on each side. At each side of the tyre a complete coil is enclosed in a pocket near the edge of the outer-cover; one half of each of the other coils is outside, and the remaining halves inside, the pocket. By this device a wire, composed of two half coils, is exposed all round between the cover and the rim. When the tyre is deflated this exposed wire can easily be pulled up with the fingers, the detached coil is then brought over the edge of the rim, more of the slack pushed back into the pocket, enlarging the other coils, whereupon the outer-cover can be removed from the rim.

Tyres with Adjustable Wires.—The '1894 Preston-Davies' tyre was attached to the rim by means of a wire running through the edge of the outer-cover, one end of the wire having a knob which fitted into a corresponding slot in the rim, the other end having a screwed pin attached to the wire by an inch or two of a very small specially made chain. This chain was introduced to take the sharp bend where the adjusting nut drew up the slack of the wire in tightening it upon the rim.

In the 'Scottish' tyre (fig. 528) the ends of the adjustable wire are brought together by a right- and left-handed screw. A short wire, terminating in a loop, forms a handle for turning the screw. When in position this handle fits between the rim and outer-cover.

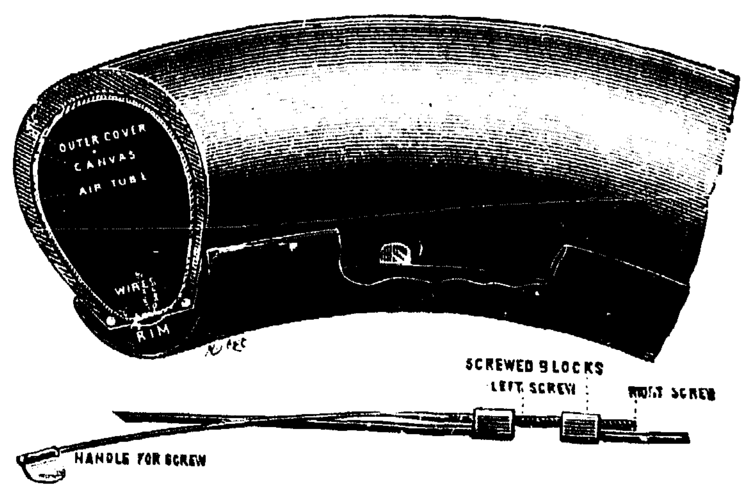


FIG. 528.

The 'Seddon' pneumatic tyre was the first successful wired tyre. Figure 529 is a view showing a portion of the tyre with the fastening released. The ends of the wire were secured by means of a small screw which was passed through the rim and locked in place by a nut. The ends of the wire were pulled together by means of a special screw wrench.

In the 'Michelin' tyre the wires are of square tubular section.

The outer-cover, which is very deep, is provided at its edges with thick beads turned outwards, and each rested in the grooves of the specially-formed rim. A tubular wire is placed round these beads

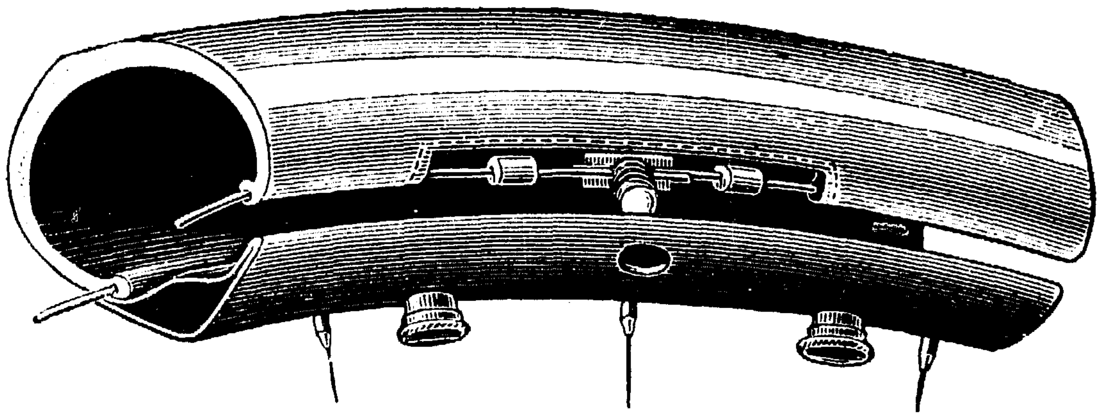


FIG. 529.

and its ends are secured in notches cut in the rim, a **T** bolt and screw securing the ends in position.

In the 'Drayton' tyre (fig. 530) the wires are tightened on the rim by a screw-and-toggle-joint arrangement.

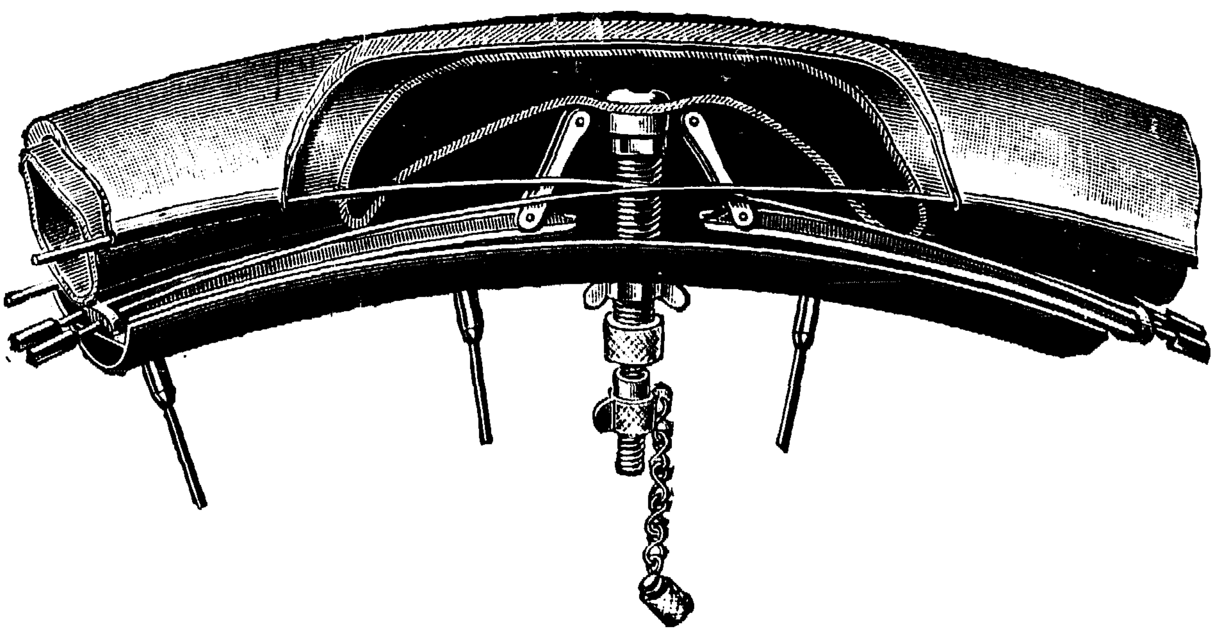


FIG. 530.

341. **Devices for Preventing, and Minimising the Effect of, Punctures.**—In the 'Silvertown Self-closure' tyre, which was of the single-tube variety, a semi-liquid solution of rubber was left on the inner surface of the tyre. When a small puncture was made, the internal pressure forced some of the solution into the hole, and the solvent evaporating, the puncture was automatically repaired.

In the 'Macintosh' tyre a section of the air-tube when deflated took the form shown in figure 531. On inflation the part of the air-tube at *S* was strongly compressed, so that if a puncture took place the elasticity of the indiarubber and the internal pressure combined to close up the hole.

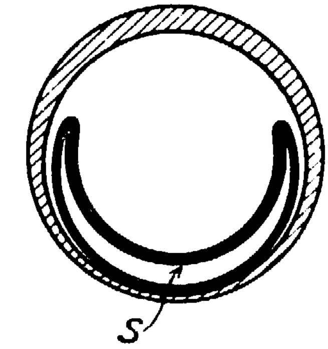


FIG. 531.

In the 'Self-healing Air-Chamber' the same principle is made use of; the tread of an ordinary air-tube is lined inside with a layer of vulcanised indiarubber contracted in every direction. When the chamber is punctured on the tread, the lining of contracted indiarubber expands and fills up the hole, so preventing the escape of air.

In the 'Preston-Davies' tyre a double air-chamber with a separate valve to each was used. If puncture of one chamber took place it was deflated and the second chamber brought into use.

In the 'Morgan and Wright Quick-repair Tyre' (fig. 532) the air-tube is provided with a continuous patching ply, which normally rests in contact with that portion near to the rim. To repair a puncture a cement nozzle is introduced through the outer casing and tread of the air-tube (fig. 533), and a small quantity of cement is left between the tread and the patching ply. On pressing down the tread the patching ply is cemented over the hole, and the tyre is ready for use as soon as the cement has hardened.

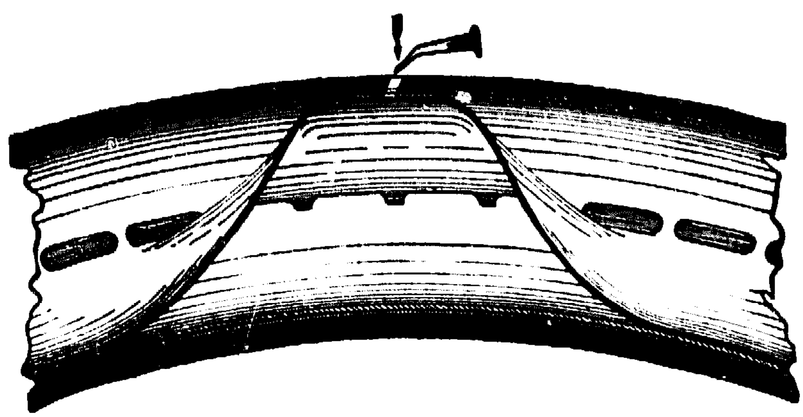


FIG. 532.

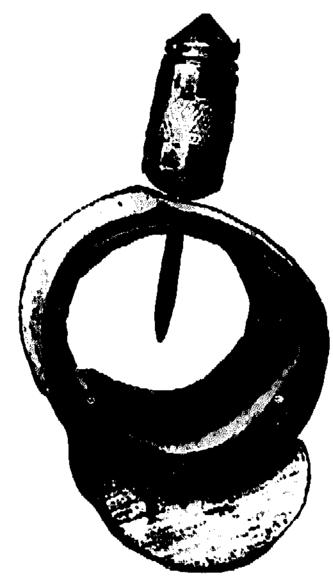


FIG. 533.

A punctured air-tube is usually repaired on the outside, so that the air pressure tends to blow away the patch. In the 'Fleuss'

tubeless tyre, on the other hand, a puncture is repaired from the inside, the tyre can be pumped up hard immediately, and the air pressure presses the patch closely against the sides of the hole.

342. **Non-slipping Covers** have projections from the smooth tread that penetrate thin mud and get actual contact with the solid ground (see sec. 170). These projections have been made diamond-shaped and oat-shaped, in the form of transverse bands, longitudinal bands (fig. 521), and interrupted longitudinal bands (fig. 527). They should offer resistance to circumferential as well as to side-slipping, though the latter should be the greater. Probably, therefore, the oat-shaped projections and the interrupted bands (fig. 527) are better than continuous longitudinal bands, and the latter in turn better than transverse bands.

343. **Pumps and Valves.**—Figure 534 shows diagrammatically the pump used for forcing the air into the tyre. The pump barrel B is a long tube closed at one end, and having a gland G screwed on to the other, through which a tubular plunger P works loosely. To the inner end of the plunger a cup-leather L is fastened. When the air-pressure in the inner part B^1 of the barrel is greater than in the outer part B, the edge of the cup-leather is pressed firmly against the sides of the barrel; but when the pressure in the space B^1 is less than in the space B the cup-leather leaves the sides of the barrel and allows the air to flow past it from B into B^1. A valve V at the inner end of the plunger allows the air to flow from B^1 through the hollow plunger and connecting tube to the tyre, but closes the opening immediately the air tends to flow in the opposite direction. The action is as follows: The plunger being at the bottom, and just beginning the outward stroke, the volume B^1 is enlarged, the air-pressure in B^1 falls, and the valve V is closed by the air-pressure in the hollow

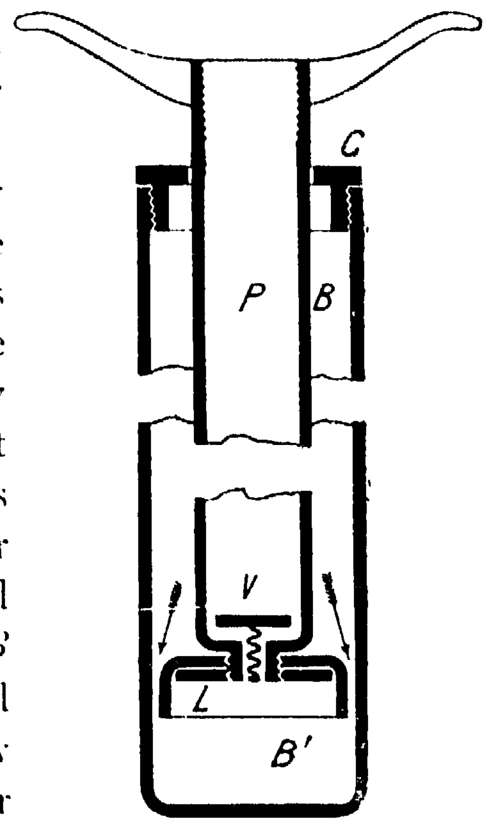


FIG. 534.

plunger (the same as that in the tyre). The outward stroke of the plunger continuing, a partial vacuum is formed in B^1, the cup-leather leaves the sides of the barrel, and air passes from the space B to space B^1, until the outward stroke is completed. On beginning the inward stroke, the air in B^1 is compressed, forcing the edge of the cup-leather against the sides of the barrel, and so preventing any air escaping. The inward stroke continuing, the

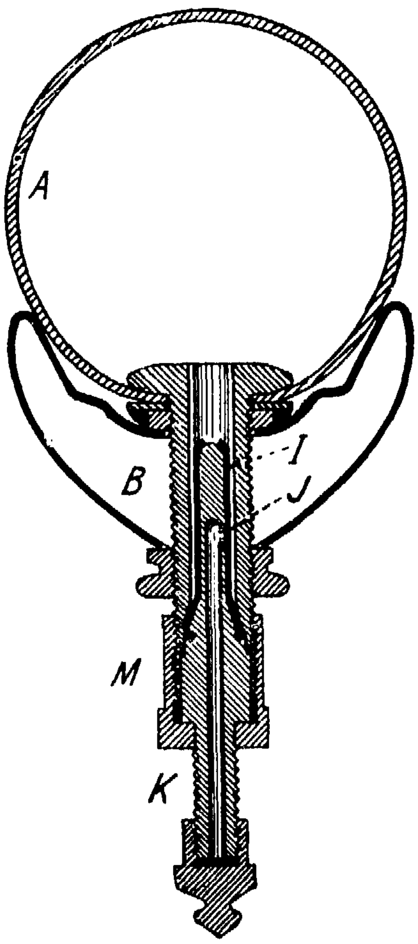


FIG. 535.

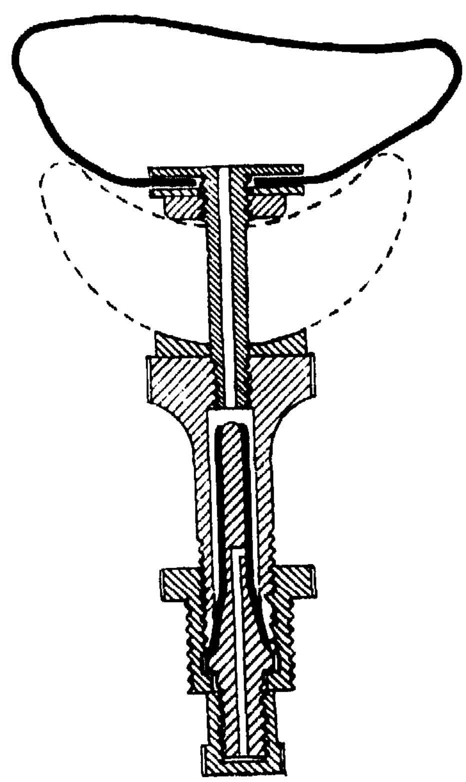


FIG. 536.

air in B^1 is compressed until its pressure reaches that of the air in the tyre, the valve V is lifted, and the air passes from B^1, along the hollow plunger, into the tyre. At the same time a partial vacuum is formed in the space B, and air passes into this space through the opening left between the plunger and the gland G.

A valve is always attached to the stem of the air-tube, so as to give connection, when required, between the pump and the interior of the tyre. A non-return valve is the most convenient, *i.e.* one which allows air to pass into the tyre when the pressure in

the pump is greater than that in the tyre, and does not allow the air to pass out of the tyre. In the 'Dunlop' valve (fig. 535) the valve proper is a piece of indiarubber tube *I*, resting tightly on a cylindrical 'air-plug,' *K*. The air from the pump passes from the outside down the centre of the air-plug, out sideways at *J*, then between the air-plug and indiarubber tube *I* to the inside of the air-tube *A* of the tyre. Immediately the pressure of the pump is relaxed, the indiarubber tube *I* fits again tightly on the air-plug and closes the air-hole *J*. By unscrewing the large cap *M*, the tyre may be deflated.

Wood rims are seriously weakened by the comparatively large hole necessary for the valve-body *B*. Figure 536 shows a valve fitting, designed by the author, in which the smallest possible hole is required to be drilled through the rim.

CHAPTER XXX

PEDALS, CRANKS, AND BOTTOM BRACKETS

344. **Pedals.**—Figure 537 shows the ball rubber pedal, as made by Mr. William Bown, in ordinary use up to a year or two ago. The thick end of the pin is passed through the eye of the crank and secured by a nut on the inner side of the crank. The pedal-pin is exposed along nearly its whole length, there are therefore four places at which dust may enter, or oil escape from, the ball-bearings.

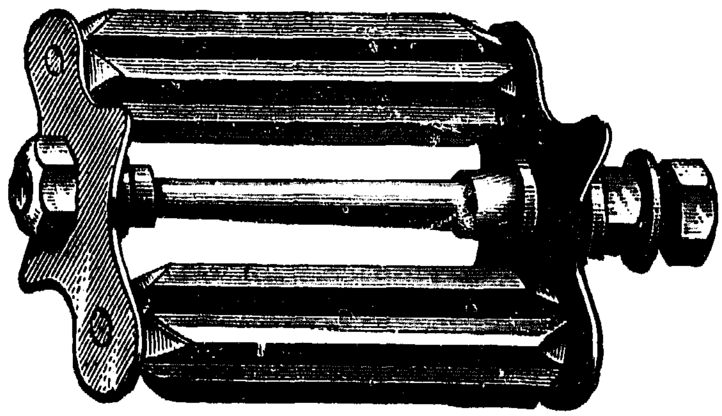


FIG. 537.

If the two pedal-plates be connected by a tube, a considerable improvement is effected, the pedal-pin being enclosed; while if in addition a dust cap be placed over the adjusting cone at the end of the spindle,

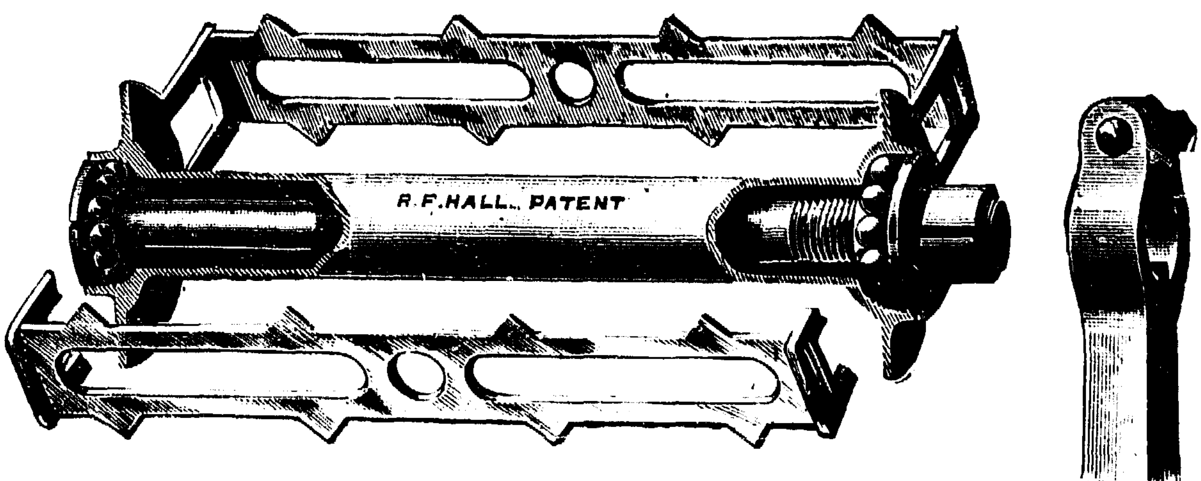


FIG. 538.

there is only one place at which dust may enter or oil escape from the bearings.

Figure 538 illustrates the pedal made by the Cycle Components Manufacturing Company, Limited, in which there are only three pieces, viz.: the pedal frame, pin, and adjustment cone. The adjustment cone is screwed on the crank end of the pedal-pin, a portion of the cone is screwed on the outside and split. The cone is then screwed into the eye of the crank, the pedal-pin adjusted by means of a screw-driver applied at its outer end; then, by tightening up the clamping screw in the end of the crank, the crank, pedal-pin, and adjustment cone are securely locked together.

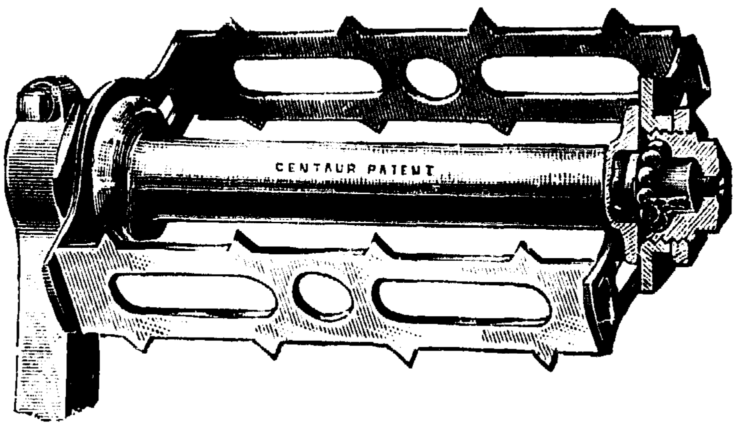


FIG. 539.

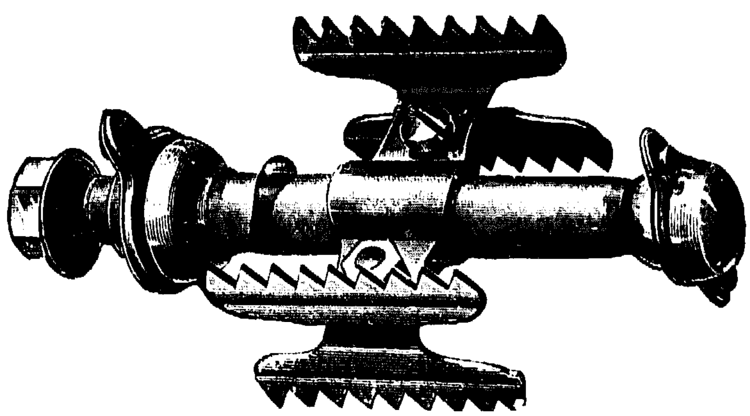


FIG. 540.

The 'Centaur' pedal (fig. 539) differs essentially from the others: the arrangement is such that an oil-bath is possible for the balls, whereas in the usual form of pedal the oil drains out of the ball-bearings.

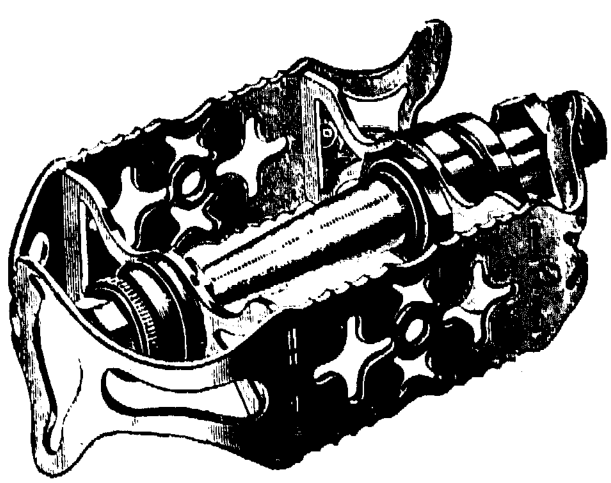


FIG. 541.

Recently, a number of new designs for pedals have been placed on the market, of which the 'Æolus Butterfly' (fig. 540), by William Bown, Limited, and that (fig. 541) by the Warwick and Stockton Company, Newark, U.S.A., may be noticed.

345. **Pedal-pins.**—The pedal-pin is rigidly fixed to the end of the crank; it may therefore be treated as a cantilever (fig. 542) supporting a load P, the pressure of the rider's foot. This load comes on at two places, the two rows of balls. One of these rows is close to the shoulder of the pin abutting against the crank, the other is near the extreme end of the pin. At any section between the balls and distant x from the outer row the bending moment is $\frac{1}{2} P x$. If d be the diameter of the pin at this section, and f the maximum stress on the material, we have, substituting in the formula $M = Z f$,

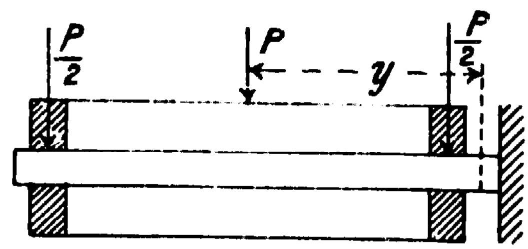


FIG. 542.

$$\tfrac{1}{2} P x = \frac{d^3 f}{10} \quad \text{or } d = \sqrt[3]{\frac{5 P x}{f}} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

That is, for equal strength throughout, the outline of the pedal-pin should be a cubical parabola. On any section between the shoulder and the inner row of balls, and distant y from the centre of the pedal, the bending moment will be $P y$.

It will in general be sufficient to determine the section of the pin at the shoulder and taper it outwards.

Example. If $P = 150$ lbs., $f = 20{,}000$ lbs. per sq. in., and the distance of P from the shoulder be 2 in., then

$$M = 150 \times 2 = 300 \text{ inch-lbs.} \quad Z = \frac{300}{20{,}000} = {\cdot}015 \text{ in.}^3$$

From Table III., p. 109, $d = \frac{9}{16}$ in.

346. **Cranks.**—Figure 543 is a diagrammatic view showing the crank-axle a, crank c, and pedal-pin p, the latter being acted on by the force P at right angles to the plane of the pedal-pin and crank. Introduce the equal and opposite forces P_1 and P_2 at the outer end of the crank, and the equal and opposite forces P_3 and P_4 at its inner end; P_1, P_2, P_3, and P_4 being each numerically equal to P. The forces P and P_1 constitute a twisting couple T of magnitude $P l_1$, acting on the crank, l_1 being the distance of P from the crank. The forces P_2 and P_3 constitute a bending couple M, of magnitude $P l$ at the boss of the crank. The force P_4

causes pressure of the crank-axle on its bearings. Thus the original force P is equivalent to the equal force P_4, a twisting couple $P\,l_1$, and a bending couple $P\,l$. No motion takes place along the line of action of P_4, nor about the axis of the twisting couple $P\,l_1$, the only work done is therefore due to the bending couple $P\,l$.

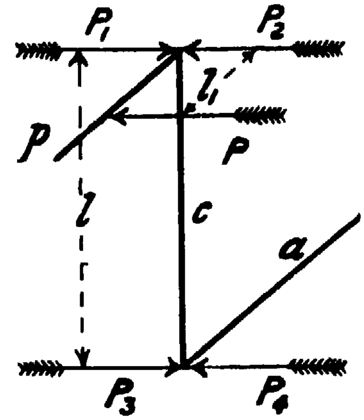


FIG. 543.

At any section of the crank distant x from its outer end, the bending-moment is $P\,x$. The equivalent twisting-moment T_e, which would produce the same maximum stress as the actual bending- and twisting-moments M and T acting simultaneously, is given by the formula $T_e = M + \sqrt{M^2 + T^2}$. Similarly, the equivalent bending-moment

$$M_e = \tfrac{1}{2}\,T_e = \tfrac{1}{2}\,(M + \sqrt{M^2 + T^2}).$$

Example.—If $l_1 = 2\tfrac{1}{4}$ in., $l = 6\tfrac{1}{2}$ in., $P = 150$ lbs., and $f = 20{,}000$ lbs. per sq. in.

$M = 150 \times 6\tfrac{1}{2} = 975$ inch-lbs., $T = 150 \times 2\tfrac{1}{4} = 337$ inch-lbs.

Then

$$T_e = 2007 \text{ inch-lbs., or } M_e = 1003 \text{ inch-lbs.}$$

Then

$$Z = \frac{M}{f} = \frac{1003}{20{,}000} = \cdot 0501 \text{ in.}^3$$

From Table III., p. 109, the diameter of a round crank at its larger end should be $\tfrac{13}{16}$ in.

If the cranks are rectangular, and assuming that an equivalent bending-moment is 1000 inch-lbs., we get

$$Z = \frac{b\,h^2}{6} = \frac{1{,}000}{20{,}000}$$

$$\therefore\ b\,h^2 = \cdot 30.$$

If $b = \tfrac{1}{2}\,h$, *i.e.* the depth of the crank be twice its thickness, we get $\tfrac{1}{2}\,h^3 = \cdot 30$, $h^3 = \cdot 60$, and

$$h = \cdot 843 \text{ in.},\ b = \cdot 421 \text{ in.}$$

The cranks were at first fastened to the axle by means of a rectangular key, half sunk into the axle and half projecting into the boss of the crank. A properly fitted and driven key gave a very secure fastening, which, however, was very difficult to take apart, and detachable cranks are now almost invariably used. Perhaps the most common form of detachable crank is that illustrated in figure 544. The crank boss is drilled to fit the axle, and a conical

FIG. 544.

pin or cotter, flattened on one side, is passed through the crank boss and bears against a corresponding flat cut on the axle. The cotter is driven tight by a hammer, and secured in position by a nut screwed on its smaller end.

In the 'Premier' detachable crank made by Messrs. W. A. Lloyd & Co. a flat is formed on the end of the axle, and the hole in the crank boss made to suit. The crank boss is split, and on being slipped on the axle end is tightened by a bolt passing through it.

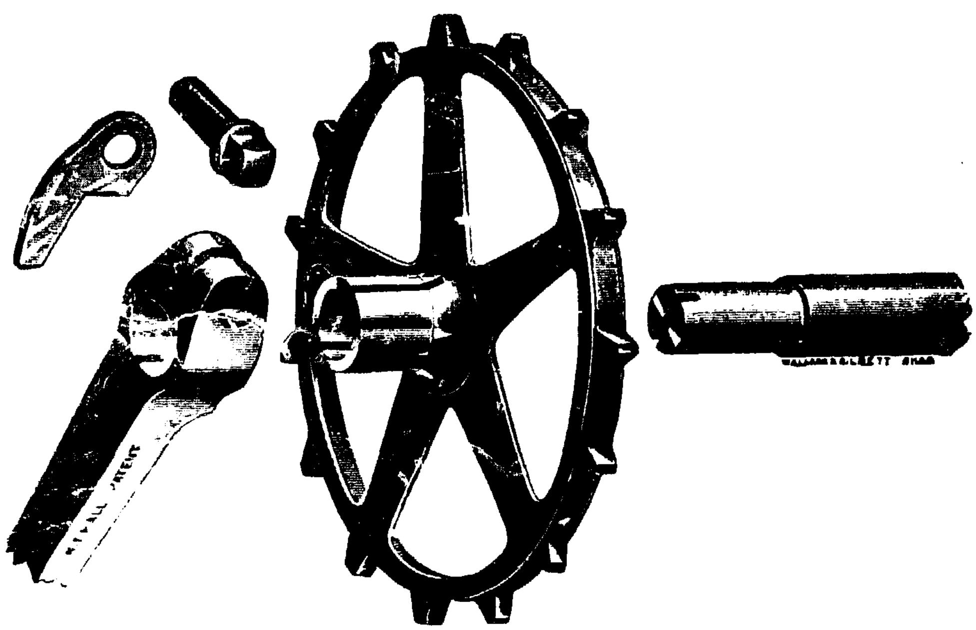


FIG. 545.

Figure 545 illustrates the detachable chain-wheel and crank made by the Cycle Components Manufacturing Company. A

long boss is made on the chain-wheel, over which the crank boss fits. Both bosses are split and are clamped to the axle by means of a screw passing through the crank boss. In addition to the frictional grip thus obtained, a positive connection is got by means of a small steel plate, applied at the end of the crank-axle and wheel boss, and retained in position by the clamping-screw. The pedal end of this crank is illustrated in figure 538.

The 'Southard' crank, which is round-bodied (fig. 544), receives during manufacture an initial twist in the direction of the twisting-moment due to the pressure on the pedal in driving ahead. The elastic limit of the material is thus artificially raised. the crank is strengthened for driving ahead, but weakened for back pedalling; as already discussed in section 123.

In the 'Centaur' detachable crank and chain-wheel, the crank boss is placed over the chain-wheel boss. Both wheel and crank are fixed to the axle by a tapered cotter, driven tight through the bosses and retained in position by a nut.

It has been shown that near the boss of the crank the bending-moment is greater than the twisting-moment. Round-bodied cranks have the best form to resist twisting, rectangular-bodied to resist bending. A crank rectangular towards the boss and round towards the eye would probably be the best. Hollow cranks of equal strength would of course be theoretically lighter than solid cranks, but the difficulty of attaching them firmly to the axle has prevented them being used to any great extent. In some of the early loop-framed tricycles, the axle, cranks, and pedal-pins were made of a single piece of tubing.

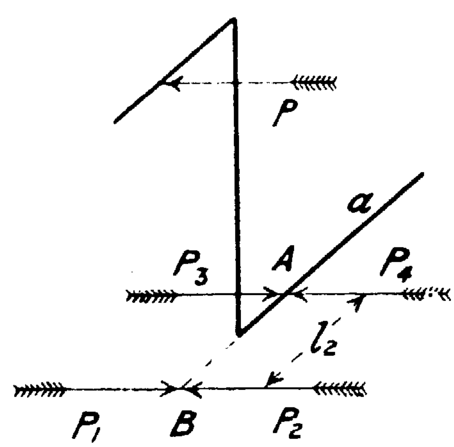


FIG. 546.

347. **Crank-axle.**—Figure 546 is a sketch showing part of the crank-axle a, the crank and pedal-pin, the latter acted on by the force P. Introduce two equal and opposite forces P_3 and P_4 at the bearing A, and two equal and opposite forces P_1 and P_2 at a point B on the axis of the crank-axle, the forces P, P_1, and P_2 lying in a plane parallel to the crank and at right angles to the crank-axle. The forces P and P_1 constitute a

twisting couple of magnitude Pl acting on the axle. This twisting-moment is constant on the portion of the axle between the crank and the chain-wheel. The forces P_2 and P_3 constitute a bending couple of magnitude Pl_2 at the point A, l_2 being the distance from the bearing to the middle of the pedal, measured parallel to the axis. The force P_1 produces a pressure on the bearing at A.

Example.—If l_2 be 3 ins., the other dimensions being as in the previous examples, $M = 150 \times 3 = 450$ inch-lbs., $T = 150 \times 6\frac{1}{2} = 975$ inch-lbs., $T_e = 1524$ inch-lbs., $M_e = 762$ inch-lbs. The diameter d of the axle will be obtained by substituting in formula (15), chap. xii., thus :

$$\frac{d^3}{5} \times 20{,}000 = 1524$$

$$d^3 = \cdot 381,\ d = \cdot 725, \text{ say } \tfrac{3}{4} \text{ in.}$$

If the axle be tubular, $Z = \dfrac{762}{20{,}000} = \cdot 0381.$

From Table IV., p. 112, a tube $\frac{7}{8}$ in. external diameter, 13 W. G., will be sufficient.

Comparing the hollow and solid axles, their sectional areas are ·226 and ·442 square inches respectively ; thus by increasing the external diameter $\frac{1}{8}$ inch and hollowing out the axle its weight may be reduced by one half ; while, if the external diameter be increased to 1 in., from Table IV., p. 112, a tube 16 W. G. will be sufficient, the sectional area being ·188 square inches ; less than 43 per cent. of that of the solid axle.

In riding ahead the maximum stresses on the axle, crank, and pedal-pins vary from zero, during the up-stroke of the pedal, to the maximum value f. If back-pedalling be indulged in, the range of stresses will be from $+f$ to $-f$. The dimensions of the axle and crank above obtained by taking $f = 20{,}000$ lbs. per sq. in. are a little greater than those obtaining in ordinary practice. A total range of stress of 40,000 lbs. per sq. in. is very high, and cranks or axles subjected to it may be expected to break after a few years' working, unless they are made of steel of very good quality. It may be pointed out here that a pedal thrust of 150 lbs. will not

be exerted continuously even in hard riding, though it may be exceeded in mounting by, and dismounting from, the pedal.

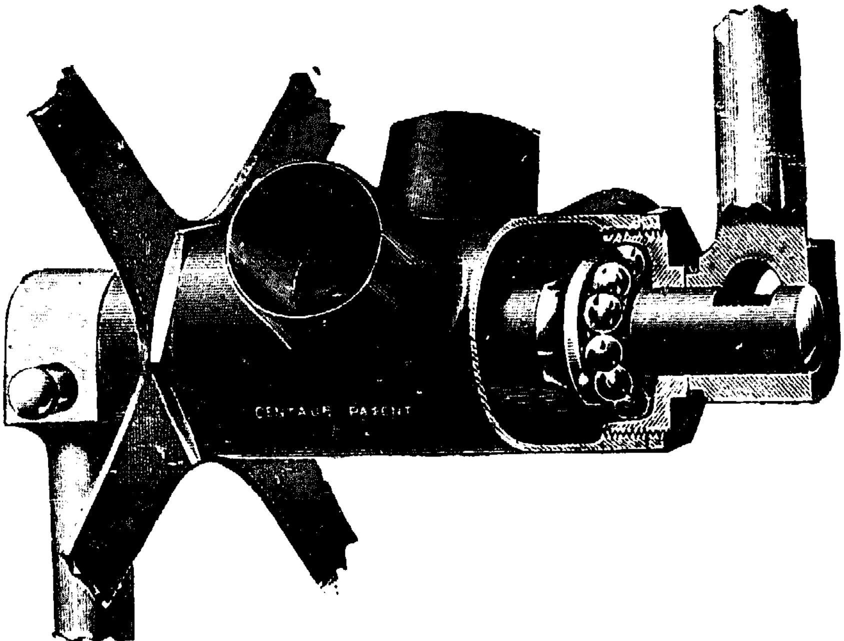


FIG. 547.

348. **Crank-brackets.**—The bracket and bearings for supporting the crank-axle form a kinematic inversion of the bearing shown in figure 404; the outer portion *H* forming the bracket is fastened

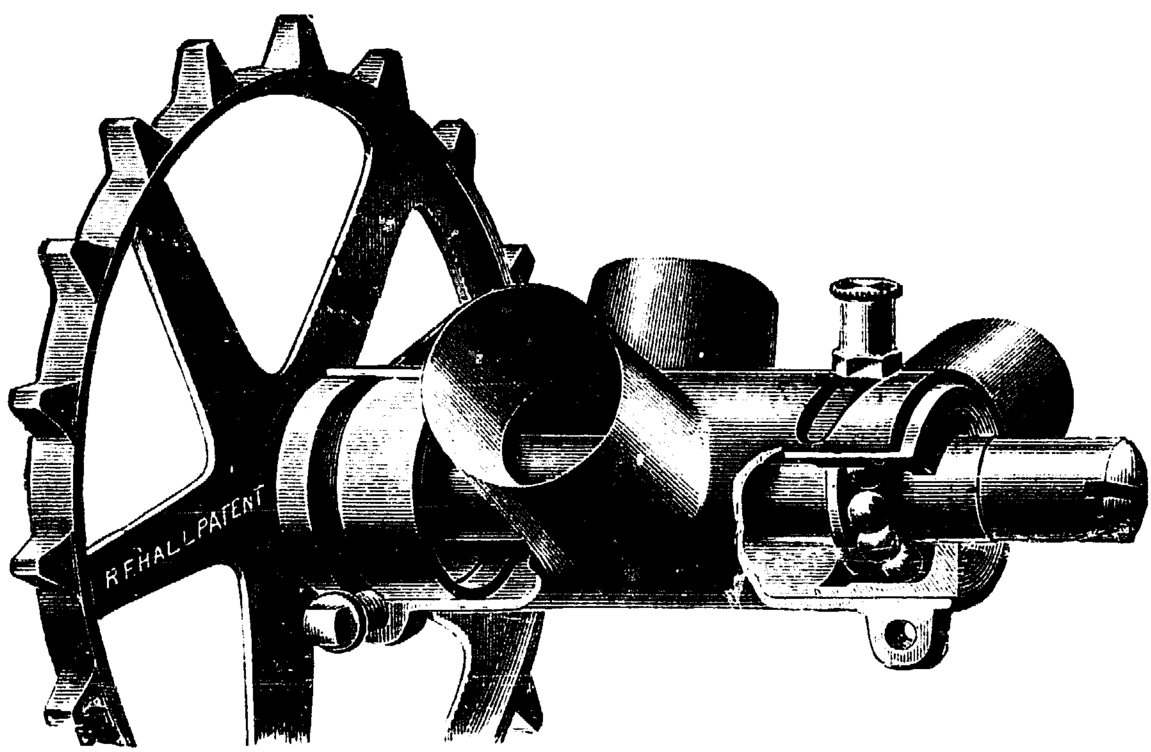


FIG. 548.

to the frame of the machine, while the spindle *S* becomes the crank-axle, to the ends of which the cranks are fastened. In the

earlier patterns of crank-brackets, hard steel cups D were forced into the ends of the bracket, and cones C were screwed on the axle, the adjusting cone being fixed in position by a lock nut.

The barrel bottom-bracket is now more generally adopted; being oil-retaining and more nearly dust-proof, it is to be preferred to the older pattern. The axle ball-races are fixed, and the adjustable ball-race can be moved along the bracket. In the 'Centaur' crank-bracket (fig. 547) the bearing *discs* or cups are screwed to the bracket, and secured by lock nuts. In the 'R. F. Hall' bracket (fig. 548) one cup is fastened to the bracket by a pin, and the other is adjusted by means of a stud screwed to the cup and working in a diagonal slot cut in the bracket. The pitch of this slot is so coarse that the adjustment is performed by pushing the stud forward as far as it will go, it being impossible to adjust too tightly. The cup is then clamped in place by the external screwed pin.

349. **The Pressure on Crank-axle bearings** is the resultant of the thrust on the pedals and the pull of the chain.

Example.—Taking the rows of balls $3\frac{1}{2}$ ins. apart, and the rest of the data as in the example of section 238, and considering first the vertical components due to the pressure P on the pedals, the condition of affairs is represented by figure 549. Taking moments about b, we get

$$3\tfrac{3}{4}P_1 = 3\tfrac{1}{2}P_3, \therefore P_3 = \frac{3\cdot75}{3\cdot5} \times 150 = 160\cdot7 \text{ lbs.}$$

In the same way, taking moments about c, we find $P_2 = 310\cdot7$ lbs.

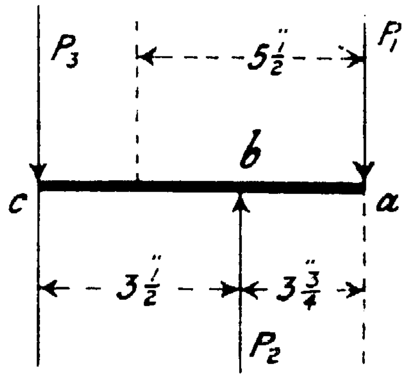


FIG. 549.

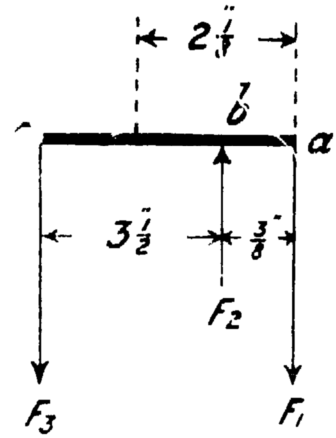


FIG. 550.

Consider now the horizontal forces. Fig. 550 represents the condition of affairs; F_1, F_2, F_3 being respectively the horizonta'

components of the pull of the chain and of the pressure on the bearings. Taking moments about *b*, we get

$$\tfrac{3}{8}F_1 = 3\tfrac{1}{2}F_3, \text{ therefore, } F_3 = \frac{\cdot 375}{3\cdot 5} \times 340 = 36\cdot 4 \text{ lbs.}$$

In the same way, taking the moments about *c*, we find

$$F_2 = 376\cdot 4 \text{ lbs.}$$

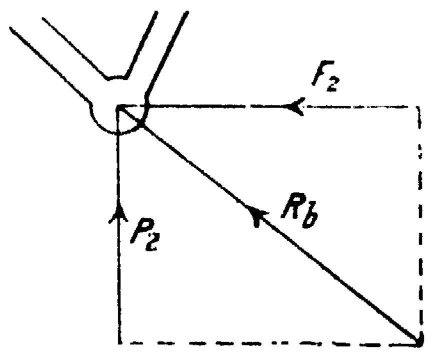


FIG. 551.

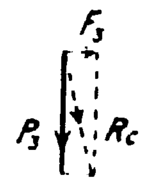


FIG. 552.

The resultant pressures R_b and R_c on the bearings *b* and *c* can be found graphically as shown in figures 551 and 552, or by calculation, thus:

$$R_b = \sqrt{P_2^2 + F_2^2} = \sqrt{311^2 + 376^2} = 488 \text{ lbs.}$$

$$R_c = \sqrt{P_3^2 + F_3^2} = \sqrt{161^2 + 36^2} = 165 \text{ lbs.}$$

CHAPTER XXXI

SPRINGS AND SADDLES

350. **Springs under the Action of suddenly Applied Load.**—We have already seen (sec. 82) that when a load is applied at the end of a long bar, the bar is stretched, and a definite amount of work is done. If the load be not too great, such a solid bar of iron or steel forms a perfect spring. If a greater extension be required for a given load, instead of a cylindrical bar a spiral spring is used. The relation between the steady load and the extension of a spiral spring is expressed by an equation similar to (2), chap. x., and the stress-strain curve is, as in figure 74, a straight line inclined to the axis of the spring.

Let a spiral spring be fixed at one end with its axis vertical (fig. 553), and let A_0 be the position of its free end when supporting no load. Let A_1 be the position of the free end when supporting a load W, the ordinate $A_1\ P_1$ being equal to W, to a convenient scale. Let $A_0\ P_1\ P_2\ P_3$ be the stress-strain curve of the spring. When this spring is supporting steadily the load W, let an extra load w be suddenly applied. The end of the spring when supporting the load $W + w$ will be in the position A_2. The work done by the loads in descending from A_1 to A_2 is $(W + w)\,x$, and is graphically represented by the area of the rectangle $A_1\ A_2\ P_2\ p_1$. The work done in stretching the spring is $Wx + \frac{1}{2}\,w\,x$, and is represented by the area $A_1\ A_2\ P_2\ P_1$.

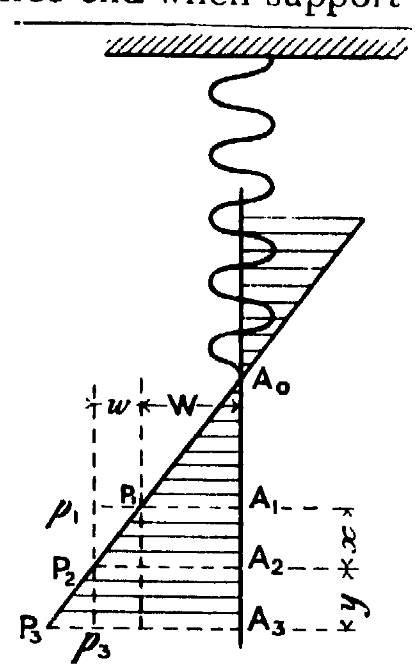


FIG. 553.

The difference of the quantities of work done by the falling weight and in stretching the bar is $\frac{1}{2} w x$, and is graphically represented by the triangle $P_1 p_1 P_2$. In the position A_2 of the end of the spring, this exists as kinetic energy, so that in this position the load must be still descending with appreciable speed. The spring continues to stretch until its end reaches a point A_3, where it comes to rest and then begins to contract. At the position of rest A_3, the work done by the loads in falling the distance $A_1 A_3$ must be equal to the work done in stretching the spring, since no kinetic energy exists in the position A_3. Therefore, area $A_1 A_3 p_3 p_1 =$ area $A_1 A_3 P_3 P_1$.

It is easily seen that this is equivalent to saying that the triangles $P_2 p_1 P_1$ and $P_2 p_3 P_3$ are equal, and therefore $y = x$; *i.e.* a load suddenly applied to a spring will stretch it twice as much as the same load applied gradually.

In the position A_3, the tension on the spring is greater than the load supported, and therefore the spring begins to contract and raise the load. If the spring had no internal friction it would contract as far as the original position A_1, and continue vibrating with simple harmonic motion between A_1 and A_3; but owing to internal friction of the molecules (or *hysteresis*) the spring will ultimately come to rest in the position of equilibrium A_2, and therefore the work lost internally is

$$P_1 p_1 P_2 = \tfrac{1}{2} w x \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

For a stiff spring the slope of $A_0 P_1 P_2 P_3$ is great, *i.e.* the extension x corresponding to a load w is small, and therefore the work lost is also small. For a weak spring the slope $A_0 p$ is small, and for a given load w the extension x, and therefore work lost, is large. But for a given extension x the work lost with a stiff spring is greater than with a weak spring.

351. **Spring Supporting Wheel.**—The function of a spring supporting the frame of a vehicle from the axle of a rolling wheel is to allow the frame to move along in a horizontal line without partaking of any vertical motion due to the inequalities of the road. This ideal motion would be attained if the stress-strain curve of the spring were a straight line parallel to its axis, and distant from it W; W being the steady load to be

supported. The wheel centre would then remain indifferently at any distance (within certain limits) from the frame of the vehicle, and since the pressure of the spring in all positions would be just equal to the weight supported, no vertical motion would be communicated to the frame. With this ideal spring the motion would be perfect until the spring got to one end or other of its stops, when a shock would be communicated to the frame. A better practical form of spring would be one having a stress-strain curve with a portion distant W from, and nearly parallel to, the axis: the slope increasing at lower and higher loads, practically as shown in figure 554.

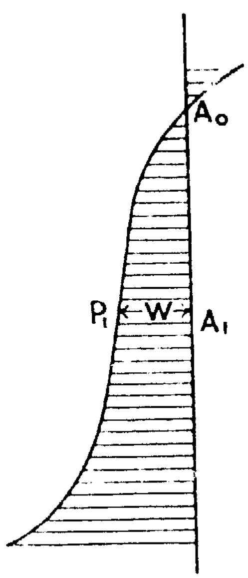


FIG. 554.

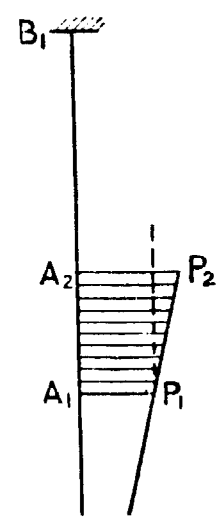


FIG. 555.

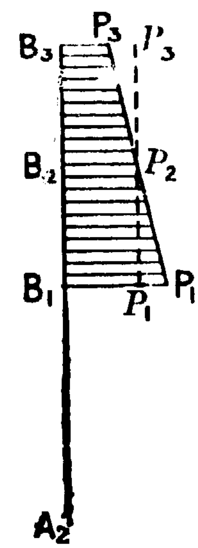


FIG. 556.

Let a cycle wheel running along a level road be supported by a spring under compression, the steady load on the latter being W, A_1 and B_1 (fig. 555) being the steady positions of the ends of the spring at the wheel axle and frame respectively. Let the wheel suddenly move over an obstacle so that its centre is raised the distance $A_1 A_2$, and the spring is further compressed. The frame end B of the spring may be considered fixed, while the wheel-centre is being raised. The work $A_1 P_1 P_2 A_2$ is expended in compressing the spring. The end A_2 may now be considered

fixed, and as the pressure on the spring is greater than the load supported, the end B will rise and lift the frame. The work $B_1 B_2 p_2 p_1$ (fig. 556) is expended in raising the frame from B_1 to B_2, where static equilibrium takes place. If the wheel-centre remain at the level A_2 the difference of energy $P_1 p_1 P_2 = \frac{1}{2} w x$ is dissipated, the frame end of the spring vibrating between positions B_1 and B_3. If the wheel return quickly to its former level A_1, little or no energy may be lost. The quantity of lost energy is smaller the more nearly the stress-strain curve P is parallel to the axis of the spring; therefore a spring for a spring-frame or wheel should be long, or the equivalent. An ideal spring would have to be very carefully adjusted, as a small deviation from the load it was designed for would send it to one end or other of its stops.

352. **Saddle Springs.**—With a rigid frame cycle, the saddle spring should perform the function above described, so that no vertical motion due to the inequality of the road be communicated to the rider; practically, the vertical springs of saddles are arranged so as to make as comfortable a seat as possible. It has been shown (Chap. XIX.) that in riding over uneven roads, the horizontal motion of the saddle is compounded of that of the mass-centre of the machine, and a horizontal pitching due to the inequalities of the road. If the saddle springs cannot yield horizontally, the rider will slip slightly on his saddle.

A saddle, as in figure 557, with three vertical spiral springs interposed between the upper and lower frames will yield horizontally more than one in which the frame and spring are merged into one structure (fig. 560).

353. **Cylindrical Spiral Springs.**—Let d be the diameter of the round wire from which the spring is made; D the mean diameter, and n the number, of the coils; C the modulus of transverse elasticity; δ the deflection, and q the maximum torsional shear, produced by a load W. Then

$$\delta = \frac{8\,n\,D^3}{C\,d^4}\,W \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$q = \frac{8\,D}{\pi\,d^3}\,W \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Mr. Hartnell says that a safe value for q for $\frac{3}{8}$-inch to $\frac{1}{4}$-inch wire, as used in safety-valve springs, is 60,000 to 70,000 lbs. per sq. in. Probably cycle springs have not such a large margin of strength as safety-valve springs. If q be taken slightly under 80,000 lbs. per sq. in., the greatest safe load, W, is given by the equation

$$W = 30{,}000 \frac{d^3}{D} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

lbs. and inches being the units.

The value of C is between 12 and 14,000,000 lbs. per sq. in. If we take $C = 12{,}800{,}000$ the deflection is given by the equation

$$\delta = \frac{n\, D^3\, W}{1{,}600{,}000\, d^4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Example.—A spiral spring $1\frac{1}{2}$-inch mean diameter, made from $\frac{1}{8}$-inch steel wire, will carry safely a load

$$W = \frac{30{,}000 \times 1}{1 \cdot 5 \times 8^3} = 40 \text{ lbs. nearly.}$$

The deflection per coil with this load will be

$$\delta = \frac{1 \cdot 5^3 \times 8^4 \times 40}{1{,}600{,}000 \times 1} = \cdot 345 \text{ inch.}$$

Round wire is more economical than wire of any other section for cylindrical spiral springs.

354. **Flat Springs.**—The deflection of a beam of uniform section of span l, supported at its two ends and carrying a load W in the middle, is given by the formula

$$\delta = \frac{W l^3}{48\, E\, I} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

E being the modulus of elasticity of the material, and I the moment of inertia of the section. For steel wire, tempered, $E =$ 13,000 to 15,000 tons per sq. in. If E be taken 33,600,000 lbs. per sq. in., substituting for I its value for a circular section $\frac{\pi}{64} d^4$, we get

$$\delta = \frac{W l^3}{80{,}000{,}000\, d^4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

lbs. and inches being the units.

In many saddles the springs are made of round wire, and are subjected both to bending and direct compression. The deflection due to stress along the axis of the wire is very small in comparison with that due to bending, and may be neglected.

355. **Saddles.**—The seat of a cycle is almost invariably made of a strip of leather supported hammock fashion at the two ends,

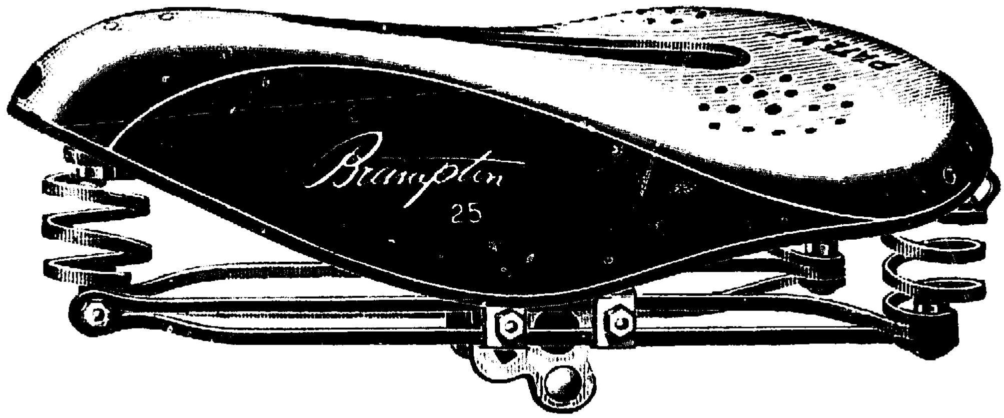


FIG. 557.

the sides being left free. In the early days of the 'Ordinary' bicycle the seat was carried by a rigid iron frame, to which the peak and back of the leather were riveted. After being in use for some time such a seat sagged considerably, and the necessity for providing a tension adjustment soon became apparent. This tension adjustment is found on all modern saddles. The iron

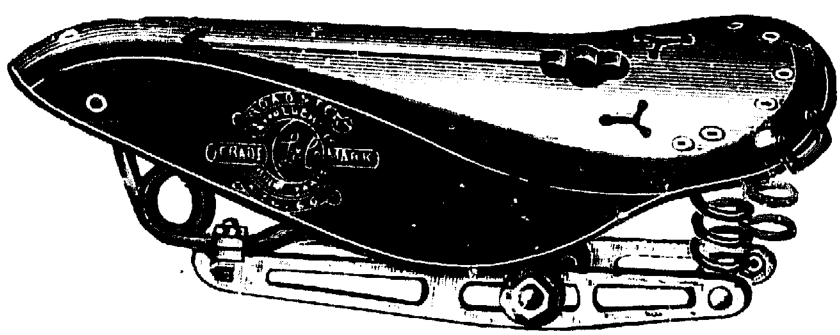


FIG. 558.

frame was itself bolted direct either to the backbone or to a flat spring, the saddle and spring were considered to a certain extent as independent parts, and were often supplied by different manufacturers. In modern saddles the seat, frame, and springs are so intimately connected that it is impossible to treat them separately.

One of the most comfortable types of saddles consists of the leather seat, the top-frame with the tension adjustment, an under-

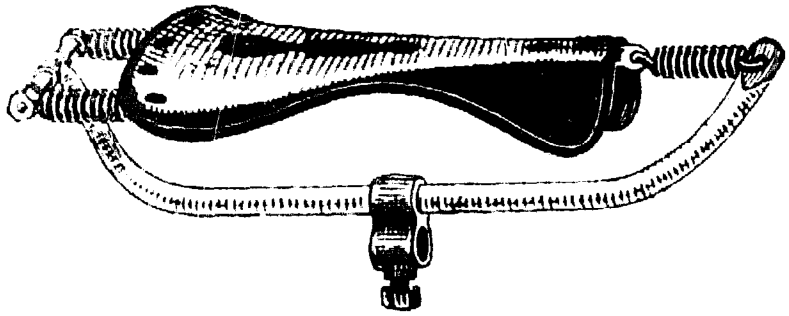


FIG. 559.

frame with clip to fasten to the **L**-pin of the bicycle, and three vertical spiral springs between the top- and under-frames.

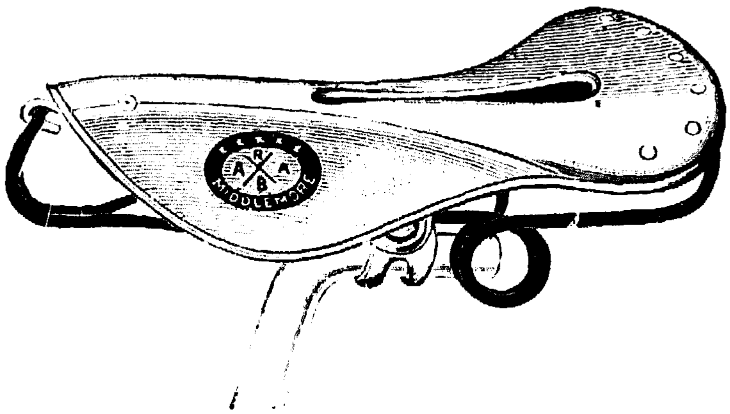


FIG. 560.

In the 'Brampton' saddle (fig. 557) the under-frame forms practically a double-trussed beam made of two wires. In

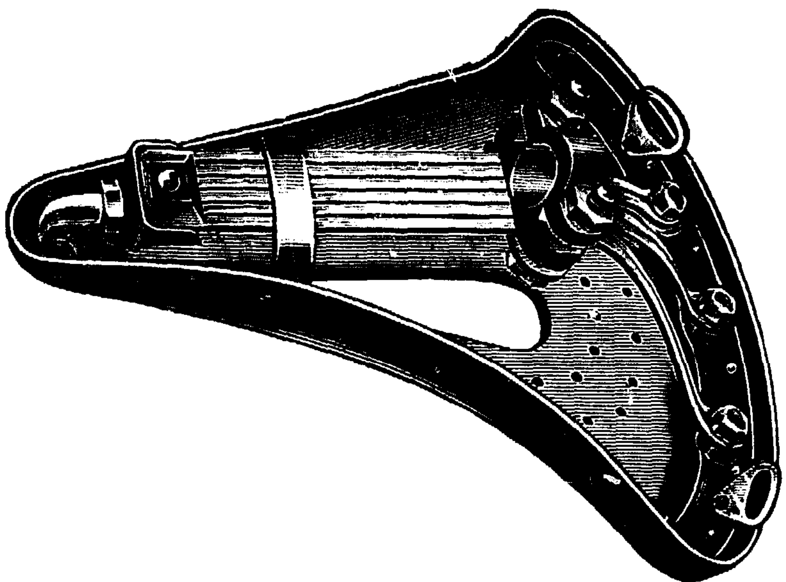


FIG. 561.

Lamplugh's saddle (fig. 558) the under-frame is made of two thin plates.

A simple hammock saddle with the seat supported by springs (fig. 559), made by Messrs. Birt & Co., consists of leather seat, tubular frame, and three spiral springs subjected to tension, no top-frame being necessary.

The springs, top- and under-frames, are often merged into one structure, as in the saddle shown in figure 560, made by

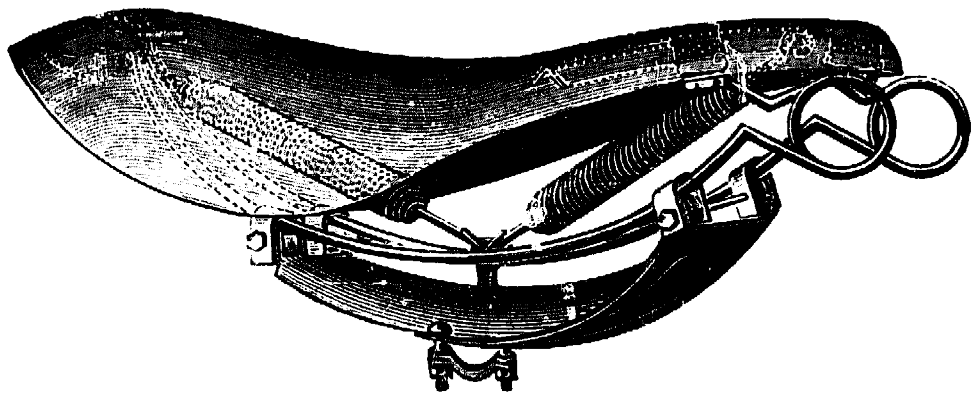


FIG. 562.

Mr. Wm. Middlemore, and that shown in figure 561, made by Messrs. Brampton & Co. In the former two wires, in the latter six wires, are used for the combined springs and frames.

All saddle-clips should be of such a form that the rider can adjust the tilt of the saddle so as to get the most comfortable

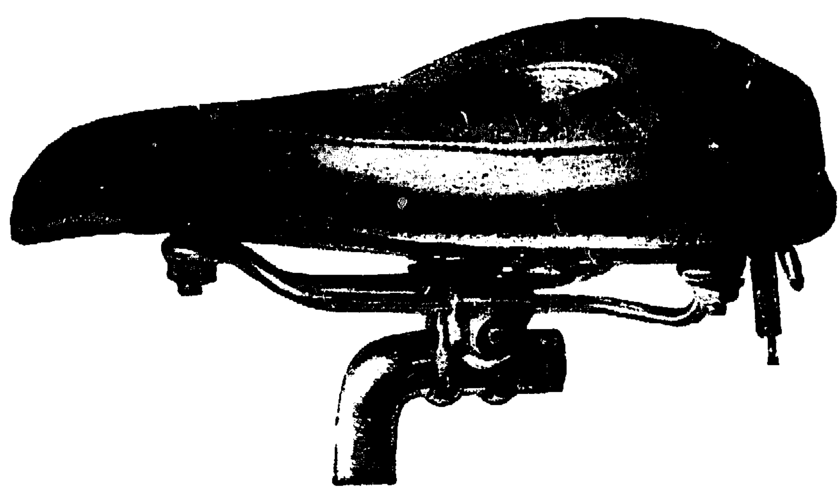


FIG. 563.

position. In the 'Automatic Cycle Saddle (fig. 562) the rider can alter the tilt while riding.

It may be noticed that the leather seats of the saddles illustrated above are slit longitudinally, the object being to avoid injurious pressure on the perineum.

The 'Sar' saddle (fig. 565), of the Cameo Cycle Company, is provided with a longitudinal depression, for the same purpose.

356. **Pneumatic Saddles.**—A number of pneumatic saddles have been made, in which the resilience is provided by com-

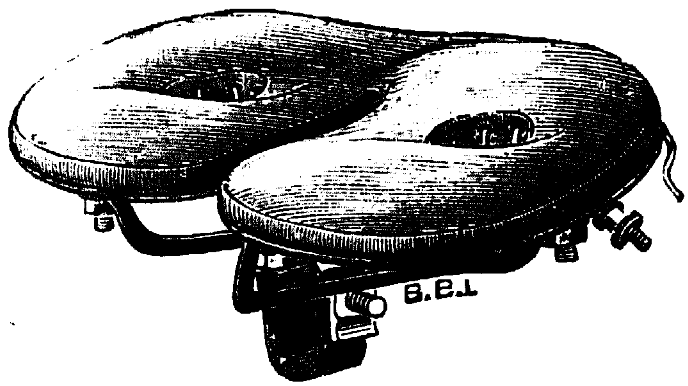


FIG. 564.

pressed air instead of steel springs. The 'Guthrie-Hall' saddle (fig. 563) is one of the most successful. The 'Henson Anatomic' saddle (fig. 564) is made without a peak, and consists of two air pads, each with a depression in which the ischial tuberosities

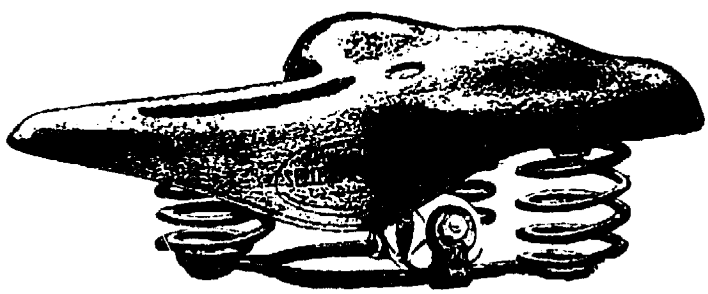


FIG. 565.

rest, the whole design of the saddle being to avoid perineal pressure. The 'Sar' saddle (fig. 565) is also provided with two depressions for the same reasons.

CHAPTER XXXII

BRAKES

357. **Brake Resistance on the Level.**—Let W be the total weight of machine and rider, W_b the load supported by the wheel to which the brake is applied, and μ_g the coefficient of friction between the ground and the tyre. If the brake be powerful enough, it may actually prevent the wheel from rotating, in which case the tyre will rub along the ground while the machine is being brought to a standstill. Then R, the greatest possible brake resistance, would be $\mu_g\ W_b$. The pressure applied at the brake handle should be, and usually is, less than that necessary to make the tyre rub on the ground; this rubbing might have disastrous results. Let v be the speed in feet per second, V in miles per hour, and l the distance in feet which must be travelled when pulling up under the greatest brake resistance. Then, since the kinetic energy of the machine and rider is expended in overcoming the brake resistance,

$$\frac{Wv^2}{2g} = \cdot 0334\ W V^2 = \mu_g\ W_b\ l,$$

or

$$l = \frac{\cdot 0334\ W\ V^2}{\mu_g\ W_b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Example.—Taking the data of the example in section 228, with the weight of the machine, 30 lbs., equally divided between the two wheels, speed 20 miles per hour, $\mu = 0{\cdot}4$, and the brake applied to the front wheel, we have $W = 180$ lbs., $W_b = 54{\cdot}3$ lbs., $R = 0{\cdot}4 \times 54{\cdot}3 = 21{\cdot}7$ lbs., and substituting in (1),

$$l = \frac{\cdot 0334 \times 180 \times 400}{0{\cdot}4 \times 54{\cdot}3} = 111 \text{ ft.}$$

If the brake be applied to the rear wheel, $W_b = 125{\cdot}7$ lbs., and

$$l = \frac{{\cdot}0334 \times 180 \times 400}{0{\cdot}4 \times 125{\cdot}7} = 48 \text{ ft.}$$

It should be noticed that the load W_b should be taken as that actually on the wheel while the brake is applied (see sec. 164).

358. **Brake Resistance Down-hill.**—If the machine be on a gradient of x part vertical to 1 on the slope, the force parallel to the road surface necessary to keep it from running downhill is $x\ W$ (see fig. 58). The brake resistance is $\mu_g\ W_b\ cos\ \phi = \mu_g\ W_b\sqrt{1 - x^2}$, ϕ being the angle of inclination to the horizontal. For all but very steep gradients, $\sqrt{1 - x^2}$ does not differ much from 1, and therefore the brake resistance is approximately $\mu_g\ W_b$, as on the level. Thus, if the brake be fully applied, the resultant maximum retarding force is $\mu_g\ W_b\sqrt{1 - x^2} - x\ W$, and therefore, as in section 357, the distance which must be travelled before being pulled up is given by the equation

$$ {\cdot}0334\ W\ V^2 = (\mu_g\ W_b\sqrt{1 - x^2} - x\ W)\ l \quad . \quad . \quad (2)$$

or

$$l = \frac{{\cdot}0334\ W\ V^2}{\mu_g\ W_b - x\ W} \text{ approx.} \quad . \quad . \quad . \quad . \quad (3)$$

If

$$x\ W = \mu_g\ W_b \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

the machine cannot be pulled up by the brake, however powerful; while if $x\ W$ is greater than $\mu_g\ W_b$ the speed will increase, and the machine run away.

Example I.—With the data of the example of section 357, brake on the front wheel, running down a gradient of 1 in 10, $x = 0{\cdot}1$; substituting in (3),

$$l = \frac{{\cdot}0334 \times 180 \times 400}{0{\cdot}4 \times 54{\cdot}3 - 0{\cdot}1 \times 180} = 643 \text{ ft.}$$

Example II.—With the same data except as to gradient, find the steepest gradient that can be safely ridden down, with the brake.

Substituting in (4), $0{\cdot}1 \times 180 = 0{\cdot}4 \times 54{\cdot}3$; or $x = {\cdot}121$. That is, no brake, however powerful, can stop the machine on a gradient of 121 in 1,000, about 1 in 8.

If the brake be applied to the back wheel, the corresponding gradient is

$$x = \frac{0{\cdot}4 \times 125{\cdot}7}{180} = {\cdot}279$$

i.e. about 1 in 4.

359. **Tyre and Rim Brakes.**—The brake is usually applied to the tyre of the front wheel, not because this is the best position, but on account of the simplicity of the necessary brake gear. In the early days of the 'Ordinary' a roller or spoon brake was sometimes applied to the rear wheel, a cord communicating with the handle-bar (fig. 338). The ordinary spoon brake (fig. 131) at the top of the front wheel fork is depressed by a rod or plunger operated by the brake-lever on the handle-bar, the leverage being about $2\frac{1}{2}$ or 3 to 1. If r be this leverage, and μ_s the coefficient of friction between the brake-spoon and the tyre, the pressure P on the brake-handle necessary to produce the maximum effect is given by the equation $\mu_s\, r\, P = \mu_g\, W_b$, or

$$P = \frac{\mu_g\, W_b}{\mu_s\, r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Example.—With the data of the example of section 357, $r = 3$, and $\mu_s = 0{\cdot}2$; substituting in (5), we get

$$P = \frac{0{\cdot}4 \times 54}{0{\cdot}2 \times 3} = 36 \text{ lbs.}$$

In the *pneumatic brake* the movement of the brake block on to the tyre is produced by means of compressed air, pumped by a rubber collapsible ball placed on the handle-bar, and led through a small india-rubber tube to an air chamber, which can be fastened to any convenient part of the frame. With this simple apparatus the brake can be as easily applied to the rear as to the front wheel.

360. **Band Brakes** are applied to the hubs of both the front and rear wheels, and have been occasionally applied at the crank-axle. The spoon brake, rubbing on the tyre, may possibly injure it; the band brake is not open to this objection. Since a small drum fixed to the hub has, relative to the frame, a less linear speed than the rim of the wheel, to produce a certain effect the brake resistance must be correspondingly larger. One end of the band is fastened to the frame, the other can be tightened by means of the

brake gear. The gear should be arranged so that when the brake is applied the tension on the fixed end of the band is the greater. If t_1 and t_2 be the tensions on the ends of the band, the resistance at the drum is $t_1 - t_2$, and, as in section 251,

$$\log. \frac{t_1}{t_2} = \cdot 4343 \mu \theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

If D and d be respectively the diameters of the wheel and the brake drum, to actually make the wheel stop revolving we must have

$$(t_1 - t_2) \frac{d}{D} = \mu_g W_h \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Example I.—Let the band have an arc of contact of three right angles with the drum, *i.e.* $\theta = \frac{3\pi}{2} = 4{\cdot}71$, let $\mu = \cdot 15$, $D =$ 28 in., $d = 5\frac{1}{4}$ in., and the rest of the data as in section 357, then, substituting in (6)

$$\log. \frac{t_1}{t_2} = \cdot 4343 \times 0{\cdot}15 \times 4{\cdot}71 = \cdot 3068.$$

Consulting a table of logarithms,

$$\frac{t_1}{t_2} = 2{\cdot}027\ ;$$

and $t_1 - t_2 = 1{\cdot}027\, t_2$. Substituting in (7),

$$1{\cdot}027\, t_2 \times \frac{5\frac{1}{4}}{28} = \cdot 4 \times 54,$$

or

$$t_2 = \frac{\cdot 4 \times 54 \times 28}{5\frac{1}{4} \times 1{\cdot}027} = 112 \text{ lbs.}$$

Example II.—If a band brake of the same diameter as in last example be applied at the crank-axle, the necessary tension t_2 will be $\frac{N_2}{N_1}$ times as great, N_1 and N_2 being the numbers of teeth in the chain-wheels on the driving-hub and crank-axle respectively. With $N_1 = 8$, $N_2 = 18$,

$$t_2 = \frac{18 \times 112}{8} = 252 \text{ lbs.}$$

This example shows the ineffectiveness of a crank-axle band brake, since the elasticity of the gear is such that the brake lever would be close up against the handle-bar long before the required pull was exerted on the band.

If oil gets in between the band and its drum, the coefficient of friction will be much less, and a much greater pull will be required, than in the above examples.

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